STRONOMY
UT MATHEMATICS.

SIR EDMUND BECKETT, BART.

ASTRONOMY

WITHOUT MATHEMATICS.

BY

SIR EDMUND BECKETT, BART.,
LL.D., Q.C., F.R.A.S.

PUBLISHED UNDER THE DIRECTION OF
THE COMMITTEE OF GENERAL LITERATURE AND EDUCATION,
APPOINTED BY THE SOCIETY FOR PROMOTING
CHRISTIAN KNOWLEDGE.

Sixth Edition,
REVISED FOR THE RESULTS OF THE TRANSIT OF VENUS.

LONDON:
SOCIETY FOR PROMOTING CHRISTIAN KNOWLEDGE.
SOLD AT THE DEPOSITORIES:
77, GREAT QUEEN STREET, LINCOLN'S INN FIELDS;
4, ROYAL EXCHANGE; 48, PICCADILLY;
AND BY ALL BOOKSELLERS.

NEW YORK: POTT, YOUNG, & CO.
1876.

LONDON:
PRINTED BY WILLIAM CLOWES AND SONS,
STAMFORD STREET AND CHARING CROSS.

PREFACE.

In the short time that has elapsed since the last edition there have been no important astronomical discoveries or events, except the long looked for Transit of Venus on 4 Dec. 1874, by which the dimensions, weights, and distances of all the solar system, beyond the moon, have been still further rectified. The first edition of this book in 1865 was the first that gave the amended dimensions, &c., which began to be adopted about that time instead of the erroneous ones deduced from the transit of 1769. And this happens to be the first to give the almost certain results (within some small fraction) of the observations of the late transit, though they have not yet been completely reduced and agreed on by astronomers.

I repeat the warning that this does not profess to be an easy book of 'popular astronomy,' because it is called 'Astronomy without Mathematics.' There

are plenty of such books, of all degrees of excellence, pictorial and descriptive, and more now than when the first edition of this came out, in a smaller form, and aiming at much less. Many persons are capable of understanding such matters as are here dealt with, who have never learnt and have no time to learn Euclid and algebra and conic sections—much less differential and integral calculus; and explanations in mathematical language and symbols would be of no use to them: though it may be true that to those who could understand them they would be easier than some parts of this, or any book which aims at teaching as much of Physical Astronomy,* or the astronomy of causes and effects, as is possible in this way. Sir G. Airy's 'Gravitation' was written on the same principle forty years ago, and has justly maintained its reputation, though the same remark has often been made, that 'it is no easier than mathematics'—i. e., to those who know them, and for whom it was

* This long established term must not be confounded with one of much narrower meaning, the 'Physics of Astronomy,' which was invented by a few persons who wanted to have a new observatory established at the public expense, for what they now call 'the Endowment of Research.' Without attempting to define it for them, they practically meant a separate establishment chiefly for spectroscopic observation of the sun, which can be and is done as well or better at Greenwich and other observatories already existing.

not written. The same may be said of parts of Herschel's admirable 'Outlines of Astronomy;' which has for some years now required complete revision to maintain its character, and should no longer be patched up with notes at the end of the stereotyped text, as all the late editions have been.

We should however soon come to a standstill in explanation if we were not allowed to use some simple propositions which can only be proved by mathematics far from elementary: such as, that a globe attracts as if it were condensed into its centre —that the time of performing an elliptic orbit is the same as of the circle which contains it—the relation of the time of pendulums to gravity, and so forth. Perhaps some readers may be hereby tempted to learn how to prove such propositions for themselves.

In one sense no elementary treatise on astronomy can be original; but no sentence in this is taken from any other; and some things here are not to be found in any other book, so far as I know, and others are in a different form from usual.

I have done the best I could to avoid mistakes, both of writing and printing, though I cannot expect to have entirely succeeded. Even great astronomers do not always escape slips of memory or of writing, hasty and erroneous conclusions from overlooking something of importance, and sometimes they have

to recant absolute mistakes of reasoning and calculation; besides the legitimate errors of imperfect knowledge, which will always be subject to correction while 'many shall run to and fro and knowledge shall be increased.'

E. B.

33 Queen Anne Street, W.
June 1876.

CONTENTS.

CHAPTER I.

CHAPTER II.

CHAPTER III.

CHAPTER IV.

CHAPTER V.

CHAPTER VI.

ASTRONOMY

WITHOUT MATHEMATICS.

CHAPTER I.

THE EARTH.

FOR several thousand years people supposed that the earth was a great platform surrounded by the sea to an unknown distance; and that the sun set in the sea and rose out of it. At least the old Greek poets said so, and invented wonderful contrivances for carrying the sun round or under the earth in the night from west to east again. Whether they really believed in such ideas or not, they certainly knew nothing of the real shape of the earth or the construction of the universe.*

But you may ask, did not David, who lived a good while before Homer, speak of the 'round world' several times in the Psalms? No, he did not. There is no such word as 'round' in the original Hebrew, nor in our Bible version of the Psalms. 'The round world' is merely a translation of the common Latin phrase for the earth, *orbis terrarum*, which was used in the Latin Vulgate Bible; and even that does not mean a globe,

* See Sir G. C. Lewis's 'Astronomy of the Ancients.'

but a round disc or wheel, though the Romans did know that the earth is a globe.*

It would have been contrary to the habit of the Bible to anticipate and reveal a scientific discovery, which men would make for themselves in time, and which was of no consequence to their religious faith and life. It is not contrary to the habit of the Bible, nor at all superfluous, to declare continually, wherever there is an opportunity (as we may say), that the sun and the moon and the stars, and the earth and all that is in it, did not grow of themselves, as some people fancy, but were created, or made out of nothing, by the word of God. For all the science in the world could never prove that 'in the beginning God created the heaven and the earth:' we can only know that by revelation. 'Through faith (not science, or observation and reasoning) we understand that the worlds were framed by the word of God, so that things which are seen were not made of things which do appear.' (Heb. xi. 3.) And all experience shows that men who disbelieve that, believe nothing else that is revealed. This has nothing to do with the proper interpretation of the particular words in which the successive acts of creation are described in the first chapter of the Bible. Nearly all learned men now agree that the word translated 'day' there does not always mean a day of twenty-four hours, but may mean periods of enormous length, each ended by some marked division of time or epoch.†

Thales, who was called one of the wise men of Greece, is commonly said to have discovered that the world

* Cicero, 'De Natura Deorum,' ii. 19, &c.

† See note at the end of the book.

is a round globe about 600 B.C.* Consequently men and water and all things stand all round it without falling off; and what we call 'upright' only means upright with reference to the surface of the earth or of water where we are, or in a line towards the centre of the earth; towards which all things fall or press; so that they fall in opposite directions here and in New Zealand, which you may see is very nearly opposite to England in that model of the earth with the countries painted on it which they call a *terrestrial globe.*

You may ask how we know all this. We know, first, that the earth is round, in some general sense of the word, by finding that we can actually go quite round it by sea or land in every direction, except where we are stopped by ice or mountains, or some other impassable obstacle. Secondly, we find that the sea is nowhere flat, but rises everywhere like a low round hill, so that the masts of distant ships are always seen before their hull or body. So great plains, which are *level* like the sea, are not flat like a table, but rise visibly between two people at a distance, as the sea rises between the ships. And such plains, and level surfaces of water all over the world, rise the same height in the same distance,

* Sir G. C. Lewis says there is no evidence in support of this tradition. It is of very little consequence; but on the other hand there seems a strong probability of the Chaldæans and the builders of the Great Pyramid having known much more of astronomy than the mere roundness of the earth, ages before Thales. See chapter on Chaldæan astronomy in Proctor's 'Saturn and his System,' and Piazzi Smyth on the 'Great Pyramid,' though I am far from believing all his theories about it. I have expressed my own views of the theory of its construction in a book on 'Architecture and Building.'

viz., 8 inches in a mile, and not 16 but 32 or 4×8 inches in 2 miles, 9×8 inches or 6 feet in 3 miles, and so on for a considerable distance; so that two very tall men 3 standing 6 miles apart on a level plain can only just see each other's heads with telescopes; and what we call the 'visible horizon,' or boundary of sight by the level ground or sea, is everywhere 3 miles from an eye 6 feet high. All this can only happen on a globe of a certain size. And what we call *level* only means flat when the surface is not large enough for the curve to be distinguished. It means really the surface which fluids take at rest, to which a plumb-line is upright, and which is equidistant everywhere from the centre of the earth—subject to a small correction which you will see at page 10.

Again we see the outline or shadow of the earth itself upon the moon in eclipses, and that is always round, whatever part of the earth may face the moon just then. Indeed, as eclipses were observed and predicted and recorded as important astronomical events long before distant voyages at sea were made, the roundness of the world was very likely first considered to be proved by them, though the notion of its being a platform may have been given up before. For a body which always casts a round shadow can be nothing but a globe, as you may easily see if you hold up things of other shapes before the sun in different positions, so as to cast their shadows on the ground.

Measuring the Earth.—After it was found that the earth is a globe, it was natural to try to measure it; but it was long before that could be done accurately. It may indeed be done approximately from the figure

just now given; for it may be proved that, if the earth is a globe, its diameter is to the distance of the visible horizon as that is to the height of your eye above the plain; which you will find gives 7920 miles, for a height of 2 yards and a distance of the horizon of 3 miles or 5280 yards. But this method admits of no great accuracy, and these figures are rather the result than the source of really accurate knowledge of the earth's size; for the rays of light near the ground are irregularly bent or refracted by the air, so that you do not in fact see straight, and cannot distinguish where the visible horizon for really straight lines of sight would be; and a very small error in the distance of the horizon will make a very large one in the size of the earth.

It has now been measured by other means, which I will describe presently; and it is found to be 24,900 miles round the *equator*; which is a circle round the middle of the earth at an equal distance from the north and south poles. The poles are the two ends of that imaginary axis round which the earth turns every day. All circles round the earth and going through the poles are called *meridians;* and so every place has its own meridian, which runs from north to south, and the sun crosses that circle at the noon or midday of that place. All circles which divide any globe equally are called *great circles,* because no greater can be drawn. Any straight cut or section through a globe which does not divide it equally, makes a *small circle.* The shortest road between any two places on a globe is by the great circle which passes through them both: hence comes what is called 'great circle sailing.' For

the arc of a great circle is less curved, or more nearly straight, than the arc of a small circle, and is therefore a shorter line between the same points, a straight line being the shortest possible. The diameter of a globe is necessarily also the diameter (or line through the centre) of every great circle; and you should remember that the circumference of every circle is very nearly $3\frac{1}{7}$ of its diameter: that is, if the diameter is 7 feet or miles the circumference may be called 22; or more exactly, circumference = 3·1416 diameters very nearly: but no number of figures can express the exact proportion. The *radius* is half the diameter.

The greatest equatorial diameter is 7926·6 miles. Some measurers of the earth have made it nearly two miles less than this through 104° east and 76° west longitude; but later calculations seem to make this doubtful. At any rate we may treat the earth as 7926 miles wide at the equator, and all the sections parallel to it as circles, which are called *parallels of latitude.*

The polar axis is estimated at $7899\frac{1}{2}$ miles, or 500 millions of inches, a thousandth part longer than our present standard inch, which probably only came by accident to be what it was when the standard was taken, and might just as well be a thousandth more. For the other European nations have inches too, and some of them are rather longer than ours. The French *metre* 39·371 inches is the worst measure in the world, because it is inconsistent with any natural one; whereas our yard is the long stride of a man of good height, and the natural length of his walking-stick, and half his height or half the stretch of his arms. And the

metre is not even what it pretends to be, the 40 millionth of a meridian of the earth; for the measure taken was erroneous; and if it were, such a standard is of no more real value than the distance of the moon. Yet there are people who have engaged in the crusade of trying to force on us this bad, erroneous, arbitrary and revolutionary measure of a nation which tried also to abolish the week and make a new one of ten days, and whose language is declining over the world, while ours already prevails over more regions of the world than any other, and is evidently destined to advance more and more.*

The polar axis being thus about 26½ miles less than the equatorial one, the earth is not quite a globe or sphere, but what is called a *spheroid;* which means something like a sphere. There are two kinds of spheroids, one flattened at the poles, and fatter round the equator, as the earth is, which is called an *oblate spheroid,* and is formed by turning an ellipse round its smallest diameter; and the other, formed by turning an ellipse round its greatest diameter, is thinner at the equator, and drawn out at the poles, like an egg with two small ends, which is called a *prolate spheroid.* The spheroidicity of the earth or any other planet is usually called its ellipticity: which means the proportion which the difference of the two axes or semi-axes of an ellipse bears to the greater of them; or the proportion of EB to AC in the figure at p. 48.

Consequently every meridian of the earth is an ellipse, and not strictly a great circle; though for

* See Sir J. Herschel's 'Familiar Lectures, Celestial Weighings and Measurings,' strongly condemning the Metre as a standard.

ordinary purposes it may be called one, as the ellipticity, or the proportion of 26½ miles to 7926, is only one 298th, so little that you could not perceive it on any globe that could be made. An ellipse is shorter than the circle containing it by very nearly half the ellipticity, so long as that is small: when it is not, the relation between them is complicated. Therefore a meridian is a 596th shorter than the equator. In giving the mean velocity of the earth and planets in their orbits, which are all elliptical, I shall treat them as circles for simplicity, as their ellipticity is very small, and their periods are exactly the same as if the orbits were circular with the same mean distance from the sun.

Before explaining what *latitude* and *longitude* are, we must observe that every circle in the world has for ages been considered to consist of 360 equal parts, or arcs of its circumference, called degrees; and again, every degree (1°) contains 60′ (minutes); and every minute (1′) contains 60″ (seconds), which have nothing to do with minutes and seconds of time. That is the way that parts or arcs of circles are always measured, and angles also, or the opening between the two straight lines called radii, reaching from the centre to the circumference of any circle, whether the circle is actually drawn or only imagined to be drawn. For instance, the angle 90° means the opening at the centre of the circle between two lines drawn to the two ends of a quadrant of the circumference; and 90° is called a *right angle*, and lines at right angles are also said to be *perpendicular* to each other; for 'perpendicular' in mathematics does not always mean upright, though

an upright stick is of course perpendicular to a piece of level ground or the surface of water, which is always level. If you stick a pencil with the middle of its flat end upon a globe, it is perpendicular to the surface of the globe, or upright.

Angular Measure.—We shall often have to deal with the numerical value of angles, and that is not so many degrees or minutes, which are merely conventional quantities, but it is the proportion of the corresponding arc or piece of circumference to the length of the radius; or the angle between two radii is the arc divided by the radius. The length of an arc of 180° is 3·1416 times the length of the radius; or calling the radius 1 for simplicity as usual, 180° is 3·1416; which number occurs so often that mathematicians always use the Greek letter π (called *pi*) for it. Therefore 1° is ·0175 and 4° = ·07, which is easy to remember, and the angle whose numerical value is 1 is about 57° 18′. The arc of a *small* angle differs so little from its chord, or the straight line joining its ends, that we may practically say that the value of the angle between two lines CA CB of equal length is the distance AB between their two open ends divided by either of the lines, so long as the angle is a small one.

Latitude is the number of degrees measured from the equator, each pole being at 90°; and a degree of latitude is about 69 miles, with a qualification to be mentioned presently. Longitude is the number of degrees on each circle or parallel of latitude measured east or west from some given meridian, which each country chooses for itself, ours being that which goes through Greenwich Observatory. Consequently the

number of miles in a degree of longitude varies from 69·17 at the equator to 0 at the poles, and is about 43 here in latitude 51° 30′. A nautical mile or *knot* is 1′ of longitude at the equator, or about one sixth longer than a common mile.

But if we take an oblate globe instead of a quite spherical one, and divide any meridian into equal spaces for degrees of latitude, and apply a long flat-ended pencil to them and measure its inclination carefully by proper instruments, we shall not find equal degrees of inclination correspond with those equal degrees of latitude. In other words, the degrees so marked would be wrong and not true degrees of latitude, which mean the places where the inclination of such a pencil (called a *normal* to the earth's surface, or the true perpendicular to a *tangent* of the meridian) changes by 1°. And the difference is such that 1° of latitude is 68·8 miles at the equator, increasing to 69·4 at the poles. Consequently the plumb-line, or line perpendicular to the surface of water or mercury, or the direction of gravity, does not really point to the centre of the earth, except at the poles and the equator. Degrees of latitude are measured by the stars, as a great hollow spherical background for graduation; for they are so immensely distant that for purposes of this kind the earth's centre may be considered fixed, or the angular distances apart of all the stars are practically the same from all parts of the earth's orbit, though that is 184,000,000 miles wide, as we shall see farther on.

The stars are used for measuring longitude in the same way, except that only one star is requisite for that purpose, because we know that the earth turns

through 360°, or quite round, in 24 sidereal hours, and therefore any two places whose meridians are crossed by the same star at an interval of 4 minutes are 1° apart in longitude. And in this way the length of the equator is measured, being 360 times the miles corresponding to 1° of longitude there. The taking of levels accurate enough for these purposes is not so simple a matter as it seems, because a plumb-line or a fluid surface is liable to be disturbed by what is called local attraction, or the greater density of the soil, including water, on one side than the other. On great plains this is avoided, but great plains are not to be found everywhere, and allowance for this disturbance has to be made as well as it can be in measuring arcs of the meridian.

Long before anybody attempted to measure the difference between the equatorial and polar diameters, Sir Isaac Newton (who was born on Christmas Day 1642, and died in 1727) calculated what it ought to be: though calculation will not make it quite right, from our ignorance of the density of different parts of the earth. It was certainly once all fluid, like the lava from volcanoes, with all the water hanging over it as steam; and even now it gets 1° hotter for every 90 feet down a mine, or, as some observers say, at other depths; and water is hotter as it comes from greater depths. It would then take the shape of a globe, like a drop of rain, or melted lead in making shot, because the mutual attractions of the particles balance themselves in that shape only. When it began to spin it would swell at the equator and shrink at the poles, as a large elastic hoop will do if you spin it quickly round

its diameter. Newton calculated how much extra weight laid on the equator would balance the loss of weight or gravitation to the centre there, by reason of the centrifugal force arising from the spinning, which increases as the square of the velocity of rotation;* i.e., it would be four times as great if the earth turned twice as fast, and if it turned round in an hour and 25 min. people could not stand at the equator, but would be thrown off. Besides that, he had to calculate how much the attraction to the centre is altered by the alteration of the shape from a sphere to a spheroid, and the result is compounded of those two calculations (see p. 46).

MAPS.

One consequence of the earth being round is that no map of any large part of it can be correct. You cannot make a large piece of paper lie close upon a globe without crumpling the edges; therefore if the middle of a country is drawn on the map as it would be on the globe, the outsides will be drawn too large, and *vice versâ;* and the larger the country is, the more some parts of it must be enlarged beyond others, or distorted. Maps are made on various plans, some distorting the countries in one way, and some in another. The common 'map of the world,' in two flat circles, makes the equator only twice as long as the diameter of the earth, instead of $3\frac{1}{7}$ as long; or makes it only $\frac{2}{3}$ (or, more accurately, $\frac{2}{3{\cdot}1416}$) of the length of the meridians at the edges of each hemisphere, and makes the meridians themselves vary in the same proportion. Each

* Centrifugal force also increases as the diameter, but so does the attraction of a globe upon its surface, which counteracts it.

of those flat so-called hemispheres has exactly half the surface of a real hemisphere. In the usual maps of that kind the equator is equally divided for longitude; and meridians, which ought to be semi-ellipses, are drawn through each division, and then each meridian is equally divided for latitude, and curves drawn through those divisions, which again are ellipses if done properly, and those curves are the 'parallels' of latitude, though no longer parallel. Since the meridians near the edges are half as long again as the middle one, it is evident that the countries near the middle are contracted in length (north and south), though the widths are all kept right by the equidistant meridians. In order to make the areas equal, the shapes ought to be still more distorted, or the width between meridians diminished from the middle towards the edges in the same proportion as their lengths increase, i.e., the width between two meridians at a given distance close to the edges should be only $\frac{2}{3}$ of what it is at the middle.

If you want to keep the widths equal and to equalize the areas, the map must be stretched out north and south more and more as you approach the poles, and the pole of each flat hemisphere becomes an angle: you will see such maps in Mr. Proctor's little book on 'Physical Geography;' and even the whole earth represented by one map, in which the equator is made largest, and the countries near the edges horribly distorted in shape though equalized in area. These are called *equal surface* or isographic projections—a very different thing, remember, from 'isometrical perspective,' in which every square of a cube is projected into an equal rhomboid made of two equilateral triangles.

That is a particular case of what is called *orthographic* projection, which shows any part of the globe as it would be seen by lines of sight sensibly parallel, or from an infinite distance. It represents the middle of the country rightly, supposing you to be looking straight at the middle, but the outsides are crowded, and the map would be quite illegible at the edges if you attempted to take in anything approaching to a whole hemisphere.

The *gnomonic* projection supposes the eye to be at the centre of the sphere, and the required portion of it is projected by straight lines from the centre on a plane touching the sphere at the middle of the map, which has to be reversed, as it is taken from the inside of the globe, though that is right for stars. All great circles, such as equator, ecliptic, meridians, however they may lie, must now appear as straight lines, because the plane of every great circle passes through the centre of the globe where the eye is. Small circles, such as those of latitude and declination (which is the same on the celestial globe of stars as latitude in the terrestrial, or distance from the equator), appear as concentric circles if the map is polar. But if the centre of the map is not the pole, the small circles will become ellipses, of curvatures depending on their position. Some star maps are made on the supposition of the sphere being enclosed in a cube, and one sixth of it projected gnomonically on each of the six sides of the cube; but the outsides of each map are considerably widened, and so the stars appear much more thinly spread there than they really are.

If you suppose the globe transparent and the eye at

the *antipodes*, or the opposite end of a diameter to the country looked at, it is seen projected *stereographically* on any plane at right angles to that diameter, such as a plane touching the sphere at the other end of the diameter, or a glass plate parallel to that and nearer to the eye. This is in fact a perspective view of the country from the middle of its antipodes; for perspective only means transparent, and the glass plate theory is the foundation of all perspective drawing. But the country must be reversed for the map, as it is seen from the inside instead of the outside of the globe. This method has the advantage of preserving the shape of every part; but the middle parts are now more crowded than the outsides, though not so much as the converse in the orthographic projection.

The *equidistant* projection is an improvement on the stereographic; the eye is raised above the sphere, still considered transparent, ·707 of the radius (which is the proportion of a side to a diameter of a square, and of half the chord of a quadrant to the radius). If you draw a circle standing on a tangent, which would be the vertical section of a sphere standing on a table, and divide the lower part into any equal divisions, and draw lines through them from a point ·707 above the circle, you will find that those lines cut the tangent at very nearly equal distances. Therefore if a map of stars for some considerable distance round the pole star is made in this way, they will all be represented practically at the right distance from the pole star; but still they will be rather widened out in their distances from each other towards the edge of the map; for this is not quite an isographic projection, though very

nearly so up to (about) 40° from the centre of the map. It is used in Mr. Proctor's star maps, in which the heavens are divided into 12 circles, each comprising one of the 12 pentagons into which a sphere is divisible. Two of these embrace 38° round each pole, and 5 lie north of the equator, and 5 south, all overlapping a little. The superiority of this division is clearly shown by him, but is beyond the scope of this book. It is worth while to cut out in card a pentagon surrounded by 5 others, and make a pair of them, and then double them up into two dishes, and see how they would fit together over a sphere, making a dodecahedron, or solid of 12 equal faces.

In fact isographic maps may be made in many ways, for you may always contract in one direction as much as you increase in the other. It is only worth while to notice one more way of making a polar map of that kind, suggested by Mr. Proctor in his 'Essays on Astronomy.' Suppose a hemispherical cup set in a hemispherical dish of twice the diameter, both transparent. Spots or horizontal circles of the cup are to be projected on the dish by lines from the centre of the larger sphere, and those projection-spots or circles are again to be projected orthographically, or straight down on the table on which the dish stands. The area or ring between any two of the flat circles will equal the corresponding ring of the cup, the flat rings getting narrower as they get larger. In fact not merely a hemisphere but a whole sphere may be so represented, but the distortion becomes intolerable when the rings increase in the map while they diminish on the sphere.

In *Mercator's projection*, which is a favourite one for maps, the globe is supposed to be stretched out on the inside of a cylinder which touches it all round the equator, and the cylinder is then cut and opened out flat or 'developed.' But besides that, since *parallels* of latitude in England would be stretched wider to fit the cylinder, in about the proportion of 43 to 69 (p. 10), therefore *degrees* of latitude or the lengths of pieces of the meridian are drawn in such maps wider than those near the equator in the same proportion, in order to keep the true proportions between the length and breadth of each division of the map; and the dimensions increase still faster towards the poles; for this reason it is unfit for maps near the poles, and the maps of countries of high latitude must be made on a different scale from those near the equator; or rather, as if they had been developed from a different globe, in order to get them on anything like the same scale.

A modification of Mercator's projection may be made isographic, but again distorting the shapes in order to equalize the areas. If a sphere is surrounded by a cylinder, each band or *zone* of the sphere equals the corresponding band of the cylinder, viz., the band included between the same levels. Therefore a zone, or any country of it near the poles, would be very much widened east and west, and shortened north and south, but its area would be correctly represented on the cylinder.

There is yet another, very convenient for some purposes, called the *conical* projection. Suppose you want to map a country in the latitude of England. A hollow cone is supposed to be dropped over the globe, of such

an angle that it will touch it all round at latitude 52°, and therefore the top of the cone will be vertically over the north pole. Then the country is drawn as it would appear on the inside of this cone to an eye at the centre of the earth, and the cone is 'developed.' Consequently the meridians are all straight lines converging towards a point which was the top of the cone, and the parallels of latitude are nearly equidistant, and in fact are drawn quite so for convenience. In this there is scarcely any distortion for a moderate breadth of country from north to south, or a *zone* between two parallels of latitude near to the circle of contact with the cone. This also is used for star maps, and so indeed are all the projections, except Mercator's and the orthographic, but the equidistant clearly is the best.

Sir J. Herschel remarks that London is very nearly the centre of that hemisphere of the globe which contains more dry land than a hemisphere described round any other place as its pole. Those who have read a little Greek history know that Delphi was called the *navel* of the world, being then supposed to be the middle. The real one, you see, is not in Greece, but in England. In order to see this, take a terrestrial globe, and elevate the north pole 51½° above the north side of the wooden horizon, and bring London up to the brass meridian: then all above the horizon is the hemisphere which has London for its pole or highest point, and it includes all Europe and Africa, and all Asia except a few promontories, and all North America and most of South, leaving only the rest of it, and Australia and some islands, to the other hemisphere.

The following proportions of land and water over the

globe, and the north and south hemispheres, and the five continents, with their islands, have been ascertained by weighing paper patterns of them taken from a globe. All the water is 145 million square miles, and all the land 51½; water north of equator 59, land 39; water south of equator 86, land 13; land in Asia 18, Africa 11½, North America 9, South 6½, Europe 3½, and Australia 3. The northern hemisphere has therefore three times as much land as the southern.

The earth's surface is four times the area of one of its great circles or 3·1416 times that of a square surrounding it, only that has to be reduced about a 400th for the ellipticity of a 298th; and the result is 197 million square miles. The surface of any zone, or band round the earth between two parallels of latitude, is proportionate to its thickness, neglecting the ellipticity. Therefore the surface of each hemisphere is divided equally at 30° of latitude; for a cut through there cuts the axis half-way between the pole and the equator.

The solid content of a globe is ·5236, or a little more than half, of the surrounding cube, or 4·1888 (i.e. $\frac{4}{3}\pi$) of the cube of its radius. And that of either a prolate or an oblate spheroid is also ·5236 or about $\frac{13}{23}$ of this surrounding elongated or flattened cube. The bulk or 'volume' or solid content of any spheroid bears the same proportion to the sphere which touches it all round its equator (either within or without) as their axes do; but it is not so with the surface, which follows no simple rule. Therefore the earth is a 298th less than the sphere which would contain it. But a sphere

of the same bulk as the earth would only have a diameter of 7917·2 miles; for $7917{\cdot}2^3$ = the polar axis $\times 7926^2$, which last we saw is the equatorial diameter. Therefore the earth contains 259,845,000,000 cubic miles. I need hardly say that 7926^2 and 7917^3 mean those figures squared and cubed, or multiplied by themselves 2 and 3 times respectively.

THE LAW OF GRAVITY.

It is a far more difficult thing to weigh the earth than to measure it; and yet that has been done. Newton, by what Herschel called 'one of his astonishing divinations,' hit upon the very weight for the earth, which is about the average of the modern experiments and calculations, viz., that it is $5\frac{1}{2}$ times as heavy as if it were all made of water, or half as heavy as lead, or twice as heavy as the heaviest stones we have; from which it is clear that the inside of the earth is violently condensed by the pressure of all about it, and it is commonly supposed to be fluid.

But before we can understand the earth-weighing experiments, it is necessary to have correct ideas of the law of gravity, or universal attraction, which is popularly supposed to have been discovered by Newton. But that is only true in a sense. You may see in any life of Kepler that he advanced the theory of universal attraction in 1609, 77 years before Newton published his 'Principia Philosophiæ Naturalis.' He knew that the moon's attraction causes the tides, which indeed must have been perceived long before. Horrocks, before his untimely death at the age of 22, in the year

of Newton's birth, had advanced so far as to attribute the moon's 'disturbances' to the difference between the sun's attraction on the moon and earth. The idea that attraction must vary inversely as the square of the distance, or is 4 times as great at half the distance, 9 times at a third of the distance, and so on, was also entertained before Newton, and was naturally suggested by its being the necessary law of emanations. For suppose light or heat or smell or sound to consist of something (no matter what) which radiates in straight lines from any body which produces them, then at 2 miles or 2 yards from the body the same rays will cover a surface twice as wide and twice as high, and therefore 4 times as large as at 1 mile or 1 yard; i.e., they will be spread 4 times thinner, and their effect on each spot will be 4 times weaker than before. It is true that this analogy fails when followed up, because gravity has no respect to area or position; but it was quite enough to suggest that law to those who were speculating on it.

But guessing at a theory is a very different thing from proving it. Kepler and Horrocks had no more doubt than Newton that the same force which brings down apples draws the moon out of a straight course into an orbit round the earth; only they could not prove it, or make out that the motion of the apple and of the moon towards the earth in a second of time correspond exactly as they ought if both are due to the same force, modified by the distances of moon and apple from the centre of the earth. Much less were they able to prove that the same force acting according to the same law of distance makes the earth and moon describe ellipses

round each other, and in short do all that gravitation does in producing and preserving the motions of the universe.

We have yet said nothing of the size, or rather the weight or mass, of the attracting bodies. The fundamental idea of the theory of gravity is that every atom in the universe attracts every other; and consequently the complete statement of it is that every body attracts every atom outside it in direct proportion to its mass, or the number of its atoms (all supposed to be ultimately of equal weight), and inversely as the square of the distance of the attracted atom from each atom of the attracting body. If the attracting body is a sphere, solid or hollow, its attraction is the same on anything outside it as if all its mass were condensed into its centre; but that is only a very convenient result of mathematical calculation, and is no part of the fundamental law of gravity. 'Mass,' as used in mathematics, generally means the same as weight, and not the same as bulk or volume, but only when the masses spoken of are all referred to the same centre of attraction. All things on the earth are practically equidistant from its centre, except for delicate experiments, and therefore the earth's attraction on them all is very nearly the same, and their masses are practically represented by their weights. But the mass that would weigh a ton in a spring balance on the sun's surface (428,860 miles from his centre) would weigh no more than an old sixpence in the same balance at the same distance from the earth. And remember that whenever we speak of distances in these matters, we always mean distances of centres, unless it is otherwise stated. So we may talk

of either the weights or the masses of the earth and planets, on the understanding that they are all to be weighed at the same distance from the sun.

But we have no such things as single atoms really, for all bodies are composed of innumerable atoms, and we must remember that B attracts A besides A's attraction of B. And the force with which they would compress a spring between them depends on the product, not the sum, of their masses.

For, as I said just now, the attraction of A on every atom of a body B is measured by the mass of A, shortly called A; and the pressure between A and two atoms of B is twice as much as that of one, and therefore of B atoms it is B times as much; and therefore the pressure between A and B is measured by A × B, modified of course by their distance, supposing them to be either in contact or kept apart by a rigid rod. So the weight of everything on the earth is the earth's mass × the body's mass ÷ earth's radius^2. The attraction of the earth at its surface, or earth's mass ÷ radius^2, is usually called 'gravity,' and written g when we are talking of things near the earth's surface, where g is practically constant, though subject to a little variation on account of the earth's shape. But when we get to the moon, the earth's attraction is of course greatly reduced by the distance.

Inertia.—In astronomy, however we have not pressure to deal with, but motion, and then another element comes in, viz., the *inertia* or resistance of everything to disturbance, whether from rest into motion, or from uniform motion in a straight line into a curve, or acceleration, or retardation, or stopping of that

uniform motion. While a pendulum or a stone is continually accelerated in falling and retarded in rising by gravity, or what we call its weight, which means its mass × the earth's attractive force, its inertia is also simply represented by the mass, though it might conceivably have been otherwise. Therefore, although the dead pressure between A and B at distance AB is $A \times B \div AB^2$, the attraction *of A on* B, which is called the *accelerating force* on B (disregarding A's own motion), is that same quantity ÷ the inertia of B, i.e., by B, and is therefore simply $A \div AB^2$; which you see is independent of the mass of B. And therefore it is true, paradoxical as it seems, that the earth moves the sun as much as if he were a pea, though the earth's own motion, and therefore their relative motion, is very different. Though two pounds have twice as much pressure as one, they fall no quicker, because they have also twice as much inertia; and indeed it would be absurd to imagine that two pounds could fall any quicker for being stuck together—except so far as they are affected by the resistance of the air, which alone makes a bunch of feathers or a cloud of dust fall slower than a leaden weight.

We often have occasion in astronomy to treat one of a pair of mutually attracting bodies A, B, as if it were at rest, and all the motion given to the other. And as the accelerating force on A is $B \div AB^2$, and on B is $A \div AB^2$, the relative accelerating force between them is $(A + B) \div AB^2$, which is the same as if B consisted of a single atom, and A of $(A + B)$ atoms (neglecting the one left for B). Therefore when we want to deal with the earth as at rest and give all their relative motion

to the moon, we have only to consider the moon as an empty shell and all her mass added to the earth.

You may have read that Faraday thought it paradoxical and contrary to another now universally received law, of the 'Conservation of Force,' or the constancy of the sum of all the forces in the universe, that bodies should gain in attraction as they approach, and *vice versâ.* And though mathematicians at once perceived the fallacy of the objection, I do not know where it has been specifically pointed out. He strangely forgot that bodies cannot be and cannot have been originally separated from each other without the exercise of force. The distance of everything from everything else represents the force which has at some time or other been employed by the Creator, directly or indirectly, in separating them; and a body in falling again to the earth develops just as much force or heat in striking, or would do the same amount of work, as was used in raising it; so that Sir W. Grove's law of conservation of force is not violated, but conserved, by Newton's law of gravity and distance.

Again Faraday asked in effect this question, which involves the same fallacy in another form:—If A is attracting B with some given velocity, and C is introduced close to B, or anywhere in the same direction, how can A have as much force to spare, emanating in the same direction, as will move C as much as it moves B? But who can introduce a new body into the universe? Whatever power could do that could introduce it with the same conditions as all other bodies, as to attracting and being attracted. C must have been existing somewhere from the beginning under the uni-

versal conditions of the law of gravity. And though C introduced beside B is accelerated by A as much as B is, we must remember that their mutual attraction and approach is thereby increased, for A is accelerated twice as much as before, supposing for simplicity that C = B. So there is no paradox at all.

Philosophers have naturally speculated on the cause of gravity or the manner of its action, but with no success as yet. Newton's great name is invoked to support a modern axiom, that 'a body cannot act where it is not;' or, as he more rationally put it, 'through a vacuum, without the intervention of anything else by or through which the force may be conveyed from one to another,' which he pronounced inconceivable. The modern discoveries that light and heat are identical, and consist of vibrations of an imponderable 'æther' pervading all space, and that electricity can produce them both and is identical with magnetism, which notoriously attracts, have led to conjectures that gravity might somehow be identified with them. And though mathematicians could not accept Faraday's notions as to the law of gravity, some of them have been disposed to accept that other conjecture of his, and have made ingenious calculations to show how vibrations may produce attraction. He confessed, however, that he had never been able to find a single experimental fact in support of the identity of those forces, though he never formally abandoned it; and as Newton undoubtedly maintained an erroneous theory of light, we are in nowise bound to accept his dictum unsupported by the smallest evidence on another point somewhat analogous.

There are also several serious objections to it. All bodies are in some degree elastic or compressible, and gaseous bodies especially. Therefore their atoms, and also those of any other gas which may be enclosed with them in any compressible vessel, cannot possibly be in contact, or they could not be brought closer, but are kept apart by some repulsive force, as we know they are in gases to immense distances. If so, there is a vacuum between them, whether small or large, across which both attractive and repulsive forces act; and so there is an end of the dictum that they cannot act through a vacuum; and in fact every atom does 'act where it is not,' if we conceive the atoms themselves to act at all, which again is a mere guess, and, I think, demonstrably a wrong one.

For that demonstration we must anticipate what has not been explained yet of the motion of the earth. Sir J. Herschel remarked, in his 'Familiar Lectures,' what is easily proved by mathematics, that if gravity took any sensible time to travel, the length of the year would have been very sensibly increased from the earliest astronomical dates until now; but that is not the case. Moreover, every astronomical calculation respecting the most distant planets, many times farther than we are from the sun, proceeds on the theory that gravity takes no time to travel those enormous distances, and all those calculations are verified by observation. Therefore, so far as we have any means of judging, gravity does act instantaneously. But it is self-evident that undulations or waves cannot be transmitted in no time at all, however fast we may conceive them to go; therefore (according to

all present knowledge) gravity cannot consist of undulations or vibrations.*

All those other forces which do consist of vibrations are temporary and variable, and require to be maintained, and can be diverted or changed into something else. Even what is called a permanent magnet has been made and can be unmade, though nobody knows what its force consists of while it lasts. But gravity alone is invariable, perpetual, self-maintaining, inconvertible, insensible to cold and heat, to chemical or electrical forces (which only prevail over it—when they do—as two pounds prevail over one), incapable of being increased, diminished, intercepted, diverted, as essential and fixed a quality of all bodies as their inertia or resistance to disturbance. All that we know of either of them is that they both vary as the mass of the body, and that it is a law of nature that they do.

And what is a law of nature, or indeed any other law? It is only a statement of what invariably is or is to be done. No law can enforce itself: it requires a living power to enforce it continually, and never less power to enforce than to ordain it. When we say that laws of nature are invariable, whatever we think we mean, we really do mean that they are invariably enforced, i.e., that some power exists which always makes the same effects follow the same causes, or makes the atoms which constitute the various kinds of matter in the universe always behave in the same way in the same circum-

* So far as I know, this last objection was first propounded by me in the 'English Mechanic' in February 1873. The other is not new; but I have seen no answer to either of them by those who support the transmission theory of gravity.

stances. And all that we really know of the nature, origin, or action of gravity is that there exists some power which invariably makes every atom of the universe tend to approach every other with a force varying inversely as the square of distance; the measure of it at some one distance for some one mass having to be determined by experiments as a standard or unit for calculation of all others. Not that this conclusion is peculiar to gravity, or is any less true of forces acting by vibration. Matter, which means no abstraction here, but all the atoms and lumps of matter in the universe, can only behave as it is made to do by the continued action of the living power which we call creative for want of any better name, but which means a great deal more. A body undisturbed by attractions might as well go on for ever with uniform velocity in a straight line as be at rest, for it must require as much force to stop it as to start it; but to divert it continually out of a straight course into a curved one, or to increase the velocity of a stone falling, or diminish it in rising, requires as much exercise of fresh force at every moment as at the preceding one, and therefore the action of some power, which is not the less a living power because it acts invariably. It must be remembered, however, that no speculations about the nature of gravity can affect the law of gravity, which is the foundation of the whole of what is called Physical Astronomy.

WEIGHING THE EARTH.

The Schehallien experiment. — The difficulty of weighing the earth arises from the extreme smallness of the attraction of the largest bodies that we can weigh

directly. The heaviest thing we have is platinum, of nearly 4 times the earth's density. Suppose we could make a globe of it 100 feet in diameter, which would weigh about 300,000 tons, and suppose we hung a ball by a long string only an inch off it, the string would have to be $1\frac{2}{3}$ mile long for the globe to move the ball that one inch against the attraction of the earth pulling it down; for by a well-known law of mechanics the deflection is to the length of the pendulum as the attraction of the globe is to that of the earth; and it follows from what I have said already, that any sphere attracts at its surface in proportion to its mass, i.e., $4{\cdot}1888 \times$ the cube of its radius $\div$ the radius^2, and therefore attracts simply as its radius; and the density of platinum being nearly 4 times that of the earth, the attraction of that globe is to that of the earth nearly as 50×4 to 4000×5280 (the feet in a mile); which you will easily see requires the pendulum to be $1\frac{2}{3}$ mile long for the deflection of an inch to be produced by such a globe of 300,000 tons. Such an experiment is of course out of the question, and to get any sensible deflection of even a very long pendulum, we must use the mass of a mountain, though its irregular shape involves some difficulty and uncertainty in calculating its attraction. The first experiment of that kind was performed at the Schehallien in Scotland, and it has always since gone by that name. It has been done again at Arthur's Seat, near Edinburgh,* and also at Mont Cenis, now more famous for its tunnel, and the results have all fairly agreed.

* See 'Phil. Trans. of 1856,' both for this experiment and the Astronomer Royal's, p. 36.

The nature of it is this. If two plumb-lines are hung 100 feet apart, they make an angle of 1″ with each other, because each is pointing, we may say, to the centre of the earth; and therefore at 6000 feet apart, or rather more than a mile, the plumb-lines would be inclined 1′ to each other. But if there is a great mass of mountain rising between them, that will attract each of the plumb-bobs, and draw them nearer together, because all matter attracts all other matter. The mass of the mountain can be calculated from its size and weighing specimens of the rocks which it is composed of; and it is possible to calculate how much the mountain ought to draw the plumb-bobs aside, and make them converge more than 1′, if the whole earth were of the same density as the mountain. But in fact the mountain never does attract them so much as it ought on that supposition. Therefore that supposition of the mountain being as dense as the average density of the earth is wrong, and mathematicians can calculate how much wrong, or how much the average density of the whole earth exceeds the ascertained density of the mountain, and they find that the earth must be on the whole about twice as dense or heavy as if it were all made of the same rocks as Schehallien, and about $5\frac{1}{2}$ times as heavy as water.

The Cavendish experiment, as it is always called, from its first performer in 1798, was invented by the Rev. John Mitchell, who did not live to do it. It was done again, many times over during four years, with every possible care to secure accuracy, by the late Francis Bailey, P.R.A.S., whose experiments fill the whole 14th volume of the 'R.A.S. Memoirs.' Caven-

dish's are in the 'Phil. Trans. of 1798.' It has also been done by Dr. Reitch abroad. What is called a torsion balance can be made more sensitive than any balance moving up and down, by hanging a longish rod with an equal ball at each end from its middle by a long and thin wire. On another strong cross bar there are two large globes of lead so arranged that by pulling certain ropes they can be brought up close to opposite sides of the balls, or of the glass case in which the balance is enclosed, so as to attract them both against the torsion force of the wire, and the deflection is measured. But how is the strength of the wire to be measured? By the time that the balls take to swing back again when the globes are withdrawn. Those vibrations in some cases took no less than 15 minutes each, which would be the time of a common pendulum 512 miles long, or the earth's attraction would have to be 810,000 times weaker than it is for a common seconds pendulum of 39·14 inches to swing in 900 seconds. With stiffer wires the vibrations were of course faster.

You must accept it as proved that the time^2 of a pendulum varies as its length, and that the time^2 of a pendulum of given length varies inversely as the force of gravity, i.e., as the mass of the earth (which we want to ascertain) $\div$ its radius^2. In like manner the time^2 of a torsion pendulum varies inversely as the force, i.e., the stiffness of the wire; and two balls hung in this way will swing twice as slowly as if they were half the weight, though the time of a swinging pendulum is independent of its weight, because its weight, which makes it swing, and the inertia vary together; while the weights do not move the

torsion pendulum, but only supply its inertia. Therefore though there are two globes attracting two balls and the deflection is doubled thereby, yet the result would be the same if there were only one, except that the balance could not then exist. The stiffness or torsion force of the wire is measured inversely by the deflection caused by the attraction of the globes. And as the stiffness also varies inversely as the time^2 of vibration, it follows that the attraction of each globe when the balls have come nearest to them is measured by the deflection $\div$ time^2; while the attraction of the earth is measured by $\frac{1}{\text{time}^2}$ of a pendulum of the same length as one of the arms of the balance; this being the short way of expressing that forces vary inversely as the time.^{2}

The rest is easy calculation. For the attraction of the earth is to that of the lead globes as earth's

$$\frac{\text{density} \times \text{radius}^3}{\text{radius}^2} \text{ is to } \frac{\text{density of lead} \times \text{radius}^3 \text{ of globe,}}{\text{distance}^2 \text{ of globe and ball}}$$

taking care to measure all the lengths in either feet or inches. Deflection being an angle, we must use its numerical value; and as it is a small one, if the length of the arm is d times the movement of the balls, we shall have this result for the comparative densities:

$$\frac{\text{earth}}{\text{lead}} = \frac{\text{seconds}^2 \text{ of vibration} \times d \times \text{radius}^3 \text{ of globe.}}{\text{distance}^2 \text{ of centres} \times \text{earth's radius}}$$

The arms of the balance used were not in fact so long as 39 inches, but about 2 feet, and therefore we must put $\frac{24}{39}\,d$ for d. It may be observed that since both the time^2 of vibration and the deflection vary as the weakness of the wire, that quantity time^2 $\times$ d will

be the same for all wires, so long as the globes and balls are the same. Not only have the balls to be encased in glass, but other casing was used, and the observer himself was not in the room, for fear of creating currents of air by his own heat, but worked by ropes outside, observing through a telescope in the wall. It is satisfactory that in spite of the difficulties in both kinds of experiments the results all agree within moderate limits; for the Cavendish experiments (rejecting doubtful results) ran from about 5·7 to 5·3, and those of the Schehallien class from 5·3 down to 5 at Mount Cenis; so that Newton's 'divination' of 5·5 is as likely as any to be right. And that makes the actual weight of the earth about 5842 trillions of tons (a trillion being 1 with 18 cyphers), since 36 cubic feet of water weigh a ton, or a tank 6 feet square every way holds 6 tons.

Perhaps it would be possible to perform the experiment directly with a balance of the common form. Suppose two balls of 10lbs. (70,000 grains), each fixed at the ends of a very sensitive and long scale-beam, with an index beyond each ball. It appears from the accounts of some of the finest balances that such a balance could be made to move sensibly under the addition of a 75th of a grain, about $\frac{1}{8}$ inch square of this paper. If a lead globe of 1 foot radius is brought close under one ball, and another close above the other ball, they would produce the same effect as adding a 5,280,000th to the weight of one ball (by the calculation at p. 30), which is the proportion of $\frac{1}{75}$ grain to 10lbs., if the density of lead is twice that of the earth.

Mine experiments.—We ought to notice another

attempt to determine the earth's density, not because it produced any reliable result, but because it proved positively that the earth's density increases inwards pretty quickly. It can be shown by much easier mathematics than are requisite to prove the more simple-looking proposition that a sphere attracts things outside it as if it were condensed into its centre, that a hollow sphere exerts no attraction at all on a particle P within it, or that the opposite attractions balance in all directions. For suppose innumerable straight lines drawn through P to the surface, so as to divide it all over into little squares, it follows from very simple geometry that the sides of any two opposite squares are as their distances from P, the squares being so small that each opposite pair make the same angle with the lines which join them. Therefore the areas of opposite squares are as their distances^2 from P; but their attractions are as the areas $\div$ the distances^2, and therefore they are equal. And as that applies to every opposite pair, the attractions are balanced in every direction.

Consequently gravity would be the same at the depth of a mile below the earth's surface as if the shell a mile thick were taken off, and we stood upon an earth of a mile less radius, provided the earth is of equal density throughout. And we have already seen that two globes of equal and uniform density attract at their surfaces in exact proportion to their radii. Therefore, if the outer shell of the earth were of the average density, gravity at the bottom of a mine a mile deep would be a 4000th less than at the top. It is also demonstrable by mathematics, that the time of vibration of a pendulum varies inversely as the square root

of the force of gravity. Therefore at a mile below the surface of an earth of uniform density pendulums would lose in the proportion of $\sqrt{4000}$ to $\sqrt{3999}$; which is practically an 8000th, or 1 second in 8000, compared with their rate above ground.* But in fact the Astronomer Royal found pendulums go faster down a deep colliery. If they had only kept the same rate it would have proved that the outer shell of the earth of that depth was somewhat lighter than the average density; but as they actually gained, it proved that the shell is much lighter.

The result obtained for the average density of the earth was 6·6, which is so much beyond all the other calculations that it cannot be accepted; and as it depends on estimates of the density of the outer shell all round the world, it is not to be compared in value to the Cavendish experiment which involves no such estimates, but merely careful performance.

It may not occur to every one that the earth's average density being twice as great as the heaviest rocks at its surface implies a very much greater increase inwards than if the earth were a cylinder with the top only known. There is 7 times as much of the earth beyond a radius of 2000 miles as there is within, since a globe of radius 4 is 8 times as big as one of radius 2. And if the whole earth is called 64 (4^3), there is a bulk of 37 beyond a radius of 3000 miles, and only 27 (3^3) within it. Then, since the density at any

* For when two numbers differ only by a small fraction, the difference between their squares is very little more than twice the fraction; and between the cubes three times; and between the square and cube roots half and a third of it respectively. Here the fraction is a 4000th.

depth that we can reach is only $2\frac{1}{2}$ times that of water, it is easy to calculate that if the average density of the outer 1000 miles is 3, that of the inner 3000 miles must be $64 \times 5.5 - 37 \times 3$ all divided by 27, which is more than 9. If we divide that sphere of 3000 miles radius again at 2000 from the centre, and assume the average density of the outer 1000 to be 6, its bulk being $\frac{19}{27}$ of the whole, we shall find that the density of the inner sphere must be 16, and so increasing inwards in some ratio of which we know nothing.

We may make a rough calculation of the pressure per square inch on the inner parts of the earth. The pressure on a square inch at any depth must be the weight of the thicker part of the long pyramid contained between four lines from the earth's centre to the surface, whose cross section is an inch at the place in question; and that weight is the mass of the thicker part of that pyramid above the square inch $\times$ the attraction of the sphere below it. But we do not know how the density and attraction increase. Suppose, for simplicity, that the result of the two variations is that the density above $\times$ the attraction below any depth is constant, then the pressure on a square inch 2000 miles deep is the weight of the thicker half of the pyramid 4000 miles high and 2 inches square at the base, attracted by the sphere of 2000 miles radius; and the thin half of the pyramid is only a 7th of the thick half, or an 8th of the whole, because 'similar' pyramids (or any other solids) vary as the cube of any of their dimensions. (Bodies are called similar when all their dimensions vary equally.) About $6\frac{1}{2}$ cubic feet of 5·5 density weigh a ton, as 36 feet of water do. The bulk of a

pyramid is a third of the area of its base × its height. Hence it is easy to calculate that the thick half of that pyramid weighs about 26,750 tons. Again, the pyramid whose cross section is an inch at 1000 miles from the earth's centre has a base 4 in. square, and therefore is four times as heavy as the other; and the thin end of it is only a 64th of the whole. Therefore the thick part will weigh nearly 120,000 tons; and these are the pressures per square inch at those depths from the surface, on the above hypothesis, which greatly underrates them, for the more the density increases inwards, the stronger the attraction becomes, on which all the internal pressure depends. If the earth were condensed, for instance, into a globe of half the diameter, everything on it would weigh four times as much as it does, and we could not stand. We can form no practical idea of such pressures beyond knowing that they must produce enormous heat, independently of that which was produced by the concussion of the particles which at some time or other ran together and formed this globe, which is probably fluid through most of its inside.

It must be understood however that nothing in astronomy depends on the absolute weight of the earth, except that of the sun, moon, and planets; their proportionate weights and all their motions would be just the same if we found the absolute weights to be twice or half what they are calculated at. Accurate knowledge of the earth's diameter is far more important, for on that every measure in the universe depends. A mistake in the received size of the earth stopped Newton's belief in his own discoveries for some years, because it

gave an erroneous measure of the moon's orbit, and therefore of her deviation from a straight course under the earth's attraction in any given short time, which deviation must bear a certain relation to the space fallen through by a stone or an apple in the same time if his law of gravity was true.

MOTIONS OF THE EARTH.

As soon as it was known that the earth is a globe hanging in space, one would think it was more natural to conclude that day and night and the apparent motion of the stars were caused by the rotation of the earth, than by the revolution of that innumerable quantity of stars, all keeping their distances from each other as if they were fixed in a frame.

People must have seen too that the elevation of the sun above the horizon at noon, and of the moon at midnight, is continually changing, and varies as much as 47° between summer and winter; and that the sun rises north-east and sets north-west in summer, but south-east and south-west in winter, and the moon and all stars near the ecliptic the converse of this. If the earth does not rotate, that requires a complicated spiral kind of motion for the sun and moon and stars, something like winding thread round a ball with a continual twist, only more complicated, because the twist returns every 6 months. If they had allowed the earth to rotate daily, the sun's annual and the moon's monthly motion would have been simple on the hypothesis that they both go round the stationary but rotating earth in planes not coinciding with the equator.

But nobody seems to have adopted even this latter modification of the old idea of the earth being absolutely fixed against both revolution and rotation; much less the completely true theory of both, except a very few singularly wise men, whom nobody believed. Instead of that they invented complicated schemes of 'cycles and epicycles' to account for the visible motions; which they did approximately, but broke down under more exact observations, even before telescopes were invented. Ptolemy of Egypt, the great astronomer and astrologer of the second century, invented one such scheme, which was received for 1500 years by those who believed any, even after Copernicus, a German priest contemporary with Luther, had propounded the true theory in 1541. Tycho Brahe invented another, which did allow the planets to go round the sun, but still made the sun himself go round the earth. He objected to Copernicus that if the earth rotates eastward, a stone dropped from the top of a high tower ought to fall a long way west of it. It was rightly answered that the stone partakes of the eastward motion of the tower, and therefore falls at the bottom. But that only answered the objection, and did not prove rotation.

Newton went further, and said that it must fall half an inch *east* of the plumb-line from a tower 256 feet high; for it takes 4 seconds to fall that height, and the top of such a tower moves an 80,000th faster than the bottom in this latitude, where we go above 380 yards eastward in a second. On the experiment being tried it did so, and that was the first direct proof of the earth's rotation.

The earth's rotation from west to east makes all the

universe appear to revolve from east to west, or to rise in the east and set in the west. For though the moon and planets, and the sun too (relatively to the earth), go round the earth eastward, or in the same direction as the earth rotates, yet their motions are so much slower, the moon taking a month and some of the planets many years to go round us, that they appear to go the other way hourly; though if you look at them at the same hour on successive days, you perceive their eastward motion by the stars behind them. In this hemisphere, motion from east to west is from left to right, looking south towards the sun. In the southern hemisphere generally, they have to look north at the sun, and so east to west means right to left. At the equator they have to turn north to the sun from March 21 to September 23, and south in the other half-year, with other variations of the time up to the two tropics, as you will see farther on.

The two following direct proofs of the earth's rotation were invented by Foucault. If a heavy ball is hung by a long string from the ceiling and set and kept swinging, taking care to make it oscillate in one plane and not revolve, it will be seen after some time to be swinging across the floor in a different direction from that in which it started: the reason is that the floor has revolved under it with the rotation of the earth. If such a pendulum were swung at the north or south pole, the floor would revolve under it in 24 hours: at the equator it would not revolve at all; and at intermediate places, such as England, it revolves slower than at the poles, but still enough to be visible in an hour or so. But if the ball is allowed to swing in

an ellipse, however narrow, instead of in a plane, the ellipse will wheel round in the same direction as the ball, from quite another cause. The best way to prevent that is to hang the ball away from the vertical by a thread and then burn it. I have heard people object that the slight stiffness of the string at the point of suspension must affect the pendulum a little; but if so, it would affect it the other way: the experiment succeeds in spite of the stiffness of the string. The same thing may be shown by the machine called the *gyroscope*, where a heavy disc or wheel, turning on pivots set in a ring which itself turns on other pivots at right angles to the disc pivots (like the gimbals of a ship-compass), will keep spinning in the same plane and with its axis pointing to the same star, while the frame which carries the ring moves round it with the rotation of the earth. This is the more complete experiment, and may be performed anywhere; for if the wheel is spun in any latitude, with its axis at right angles to the earth's axis, the force of rotation of the wheel will keep the plane of the ring directed to the same stars, while the outer frame turns round it on the other pivots, which will be parallel to the earth's axis.

From the earliest times to about 1500 A.D. we hear of only one definite suggestion that the sun does not go round the earth, but the earth round it, though it was also attributed to Pythagoras, but it is not known on what authority. Archimedes tells us in a book of his own, that another astronomer, Aristarchus, about 280 B.C., held the opinion that the earth goes round the sun in a circle, and that the size of that circle is quite insignificant compared with

the distance of the stars. Unfortunately for the credit of Archimedes, he entirely disbelieved it; as we shall see that much more modern astronomers have discredited other people's discoveries which were equally correct. It is true that nothing of the kind appears in the only small book of Aristarchus himself which has come down to us, 'On the Sizes and Distances of the Sun and Moon;' but there is nothing contradictory to it, and he may have discovered the motion of the earth after he had written his book on its distance from the sun.* One can hardly suppose that a man like Archimedes would take the trouble to combat the opinion of another mathematician on such an important question without knowing that he held it. It is quite certain however that Aristarchus knew the rotation of the earth, and that it is the earth's shadow that eclipses the moon.

But if Aristarchus satisfied himself that the earth moves round the sun, the world itself and even the best astronomers were not moved into believing it for nearly 2000 years more; that is, until the time of Copernicus above mentioned, who propounded at once the earth's rotation and its revolution with all the planets round the sun.

Then came Galileo, a still greater astronomer, who was born at Pisa in 1564, and died, as Horrocks did, in 1642, the year Newton was born, and invented the telescope, which gave him the power of examining their motions still more accurately; and he found other proofs of Copernicus's theory, and (as is well known)

* I take these statements from the 'Lives' of these two philosophers, and quotations therein: I do not profess to have read their books.

was imprisoned for three years by the Roman Inquisition for publishing them.* Afterwards, about 1680, Newton made the far greater discovery of the reason why the earth and the planets and the moon all move, and must move for ever as they do, founded on that law of gravitation. I will explain afterwards how that law affects them; for the present we will go on with their motions as they are.

Besides the earth's rotation, everybody now knows that it goes round the sun. But if you ask for a direct or positive proof of the earth's revolution, I know of none that can be given until we come to the 'aberration of light,' which proves the motion of the earth, not by the sun, but by the stars, as we shall see. But the impossibility of explaining the motions of the planets on any other theory is amply sufficient proof.

The earth then goes round the sun, so as to see the same stars again in a line with him, in a year of about 365¼ days—a day meaning the time of one rotation of the earth on its own axis from noon to noon, or the average time of the sun's twice passing the same meridian. But we shall have to consider the length of the year more exactly afterwards, and also of different kinds of days which are dealt with in astronomy.

If the world were habitable all round, and not divided by the two great oceans, it would be impossible to avoid a sudden break of time somewhere, which would make the same day December 31 on one side of the boundary and January 1 on the other side. For the days begin and end later as you go west, and

* Another of the early preachers of the Copernican theory, Giordano Bruno, was imprisoned twice as long, and finally burnt for it.

earlier as you go east; and if a man could sail round the earth westward, he would find that he had lost a day in his reckoning by the sun when he came home, and that he had gained one by sailing round the globe eastward, as the earth turns from west to east, and therefore all the heavenly bodies appear to go the other way, as explained at p. 41.

The earth's mean distance from the sun (that is, the average between the greatest and least distances) is 92,000,000 miles, according to the latest calculations, and therefore the whole length of the earth's path or *orbit* treated as a circle is 578,052,560 miles. If you work it out you will find that that comes to a rate of travelling through space of 65,941 miles an hour, or 18·317 miles in a second, or 80 times faster than sound or an ordinary cannon-ball goes through the air: 365·256 days being the sidereal year or a complete revolution of the earth, and the time is the same as if the orbit were a circle, which it is not quite.

The reason why we do not feel moving with this enormous velocity is, that it is practically constant, and deviates too little from a straight line for us to feel that we are going round a curve. The addition of the rotation velocity at night, and the loss of it by day, about 1000 miles an hour at the equator, is much too little to be perceived, and the change is also very gradual. Uniform motion in a straight line is as easy as rest, and as natural, requiring no force to maintain it. And as we carry the air with us more completely than in a closed railway carriage, there is no wind or resistance of air such as we feel outside the carriage.

The centrifugal force from rotation is so much over-balanced by the earth's attraction, which is 289 times as great at the equator, where the centrifugal force is greatest, that it is quite insensible to us. Nevertheless things *are* lighter there than in high latitudes. A spring balance which weighs rightly at the poles would mark 289 pounds as weighing 288 at the equator, from the centrifugal force alone.

Things weigh less at the equator from another cause besides, in a spring balance, or when swung as a pendulum, which vibrates slower there. An oblate spheroid attracts less there than at its poles, because they are nearer to the centre, though the attraction at a given distance from the centre is less in the polar direction than in any other, as we shall see afterwards.* If the earth's density were uniform, the difference would be only a fifth of the ellipticity or $\frac{1}{1490}$; but the inside is much denser than the outside, and it is calculated that the equatorial attraction from this cause is a 590th less than the polar. Therefore taking both causes together, 194 pounds at the poles only weigh 193 in the same spring balance at the equator; and between London and the equator 1000 pounds lose abeut 3, and a clock pendulum loses $2\frac{1}{4}$ minutes a day, for the time of vibration varies inversely as $\sqrt{}$ gravity.

Another objection may occur to you, as it did to those who imprisoned Galileo, that in several places in the Psalms it is said that 'the earth shall never move,' and so forth. But that has been answered by the late

* For a particle on the equator of such a spheroid is evidently farther from the whole mass on the average than in a sphere, and a particle on either pole nearer.

Dr. McCaul, who showed that the Hebrew word which is translated 'move,' really means to shake or totter;* and so those passages of the Bible, instead of contradicting the truth, were only waiting to confirm it as soon as the truth itself was discovered by the advance of science; for the stability of the universe against shocks and permanent disturbances is now proved to be a consequence of the law of gravitation. The same may be said of the famous passage, 'Let there be light,' which people used to admire merely for its poetic grandeur, and had no idea till this century that no other words would have been equally correct; for it is now certain that light is not a thing to be created, like water, but rather a state of things, like fire or noise. Still less would a mere inventor of a cosmogony, or scheme of creation, have made light older than the sun:—i.e., the sun in his present condition.

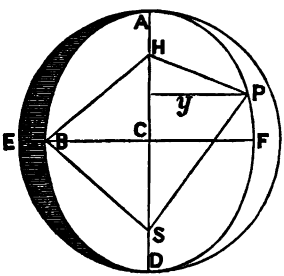
ELLIPTIC ORBIT OF THE EARTH.

Hitherto I have spoken of the earth going round the sun in a circle, as it does very nearly, but not quite; for the earth's orbit is an ellipse, and not a circle. It is remarkable that another ancient astronomer, Hipparchus, about 150 B.C., found out that the (apparent) orbit of the sun round the earth was not quite a circle, though he stopped short of the greater step which Aristarchus had probably made for himself 100 years before, and missed observing that the apparent orbit of the sun round the earth is the same as the real orbit of the earth round the sun. This figure of the

* 'Aids to Faith,' p. 219.

ellipse is so important in astronomy, that you had better learn at once what it is; for every oval or figure like a flattened circle is not an ellipse. Ovals for picture frames are often made out of four pieces of circles, two of a large one and two of a small put together; but no pieces of any circles will make a real ellipse. The simplest way to make one is to take a piece of thin string with a loop at each end; stick a pin through each loop into the table through a sheet of paper, leaving the string quite loose between them; put a pencil in to stretch the string out, and run it along, always keeping the string tight: then the pencil will describe an ellipse; and there are other ways which we need not describe beyond saying that if a straight stick has two pins and a pencil stuck in it, and the pins run along any two straight lines which cross, the pencil will trace out an ellipse.

Here is a figure of an ellipse with the circle containing it. SPH is the string; SH being the places of the pins, each of which is a *focus* of the ellipse; C the centre of both the ellipse and the circle; ACD is the *axis major* or the greatest diameter, which evidently = the length of the string or SP + HP, or twice SB. BCF is the *axis minor*. The nearer together the foci are, the more the ellipse approaches a circle, or the less eccentric it is; the proportion of CS to CA is the *eccen-*



tricity, which is therefore expressed by a fraction, either a vulgar fraction or a decimal, as may be most convenient; and you see it is much greater than the ellipticity $\frac{EB}{AC}$ (p. 8). When the ellipse is very nearly a circle the ellipticity may be called half the square of the eccentricity. Thus the eccentricity of the earth's orbit is almost a 60th, but its ellipticity is a 72,000th. CS alone may be called the linear eccentricity. We speak of the eccentricity of the planets' orbits, because we are not concerned with their centre but their focus, where the sun always is. On the other hand, we have nothing to do with the focus of a meridian of the earth, and so we speak of its ellipticity. SB or AC is also the 'mean distance,' on which the length of the year depends.

If you want to find the minor axis of a planet's orbit from the major axis and eccentricity (which are always the things given) you may do it by this rule: the square of the linear eccentricity, i.e., SC^2 = the sum of the semi-axes × their difference, or = the difference of their squares; the reason of which will be evident to any one with a little knowledge of mathematics. Taking SP alone, which is called the *radius vector*, it evidently varies faster the more eccentric the ellipse is; and we shall see the importance of that when we come to the moon's disturbances.

But though the string method is the best for drawing an ellipse, there is another definition of it which it is important to understand. An ellipse is the oblique or perspective view of a circle. For if the circle is turned a little on its diameter, it will cover from your eye the elliptic space ABDF, and all the lines at right angles

to that diameter, such as CF and y (which letter is always used in mathematical books for those lines, called *ordinates*), will be less, in proportion to the ellipticity EB, considering AC invariable. But the centre of the perspective ellipse does not coincide with that of the circle, or fall upon the line of sight from the eye to the centre of the circle, unless it is seen so far off that all the lines of sight may be considered parallel, as in looking at any of the heavenly bodies, except when we want to measure them.

The ellipse is the shape of the orbit of all the planets, and of the moons round those planets which have any. The linear eccentricity of the earth's orbit at present is ·0168, or one 60th, of its semi-axis major or mean distance of 92,000,000 miles; and therefore the earth is about 3 million miles nearer the sun at one time than another. You may very likely think this is the cause of the difference between winter and summer; but it is no such thing. On the contrary, the earth happens to be nearest to the sun in the middle of our winter, on the 1st of January, though it has not always been so, and will cease to be so again. For the whole ellipse turns round, going forwards, or in the same direction as the earth itself moves, $11''\cdot8$ a year, or 1° in 308 years; and at the same time the equinoctial points, on which the times of all the seasons depend, as you will see presently, go backwards at the rate of $50''\cdot1$ a year, or completely round in 25,868 years. And as one goes one way and the other the other way, it is the same as if the places of *perihelion* and *aphelion*, or nearest and farthest distances from the sun, went forward at the rate of $61''\cdot9$ a year, or

completely round in 20,984 years, relatively to the equinoxes, from which all celestial measures are taken, though the time of absolute or sidereal revolution of the perihelion is 109,830 years; but the motion is irregular, and the winter solstice was at aphelion 11,700 and 33,300 and 61,300 years ago.*

The time the earth takes to return to perhelion is called the *anomalistic* year, because the distance of a planet from perihelion, or of the moon from perigee or point of nearest approach to the earth, is called its *true anomaly*. The distance it would have gone in the same time if it moved uniformly, or in a circle instead of an ellipse, is its mean anomaly; and their difference is called the *equation of the centre:* all these being measured by the angles described by the radius vector round the sun—or earth in the case of the moon. The anomalistic year is 25m. longer than the equinoctial year, in consequence of the advance of the perihelion. But this is only a fact, and not a period used for calculation.

The Seasons.—The real cause of summer and winter is that the earth's axis does not stand upright in its orbit round the sun, which is called the *ecliptic*, but one pole always leans 23° 28′ towards what we call the north of the heavens or fixed stars, and the other pole leans as much to the south. Consequently, when the earth is on the south side of the sun, the north pole, and the northern hemisphere generally, are turned towards the sun, and the south pole away from him, and it is summer in the north and winter in the south. Six months after, the earth having gone half round the

* Croll on 'Climate and Time,' p. 428.

sun, the north hemisphere is turned away from him, and the south hemisphere then looks towards the sun, and so it is winter in the north and summer in the south.

The north pole being now nearest the sun in winter mitigates both the cold and heat of our climate considerably, though other causes have much to do with that, which belong rather to what is called physical geography. Sir J. Herschel gives some striking proofs that the intensity of both seasons is consequently much greater in the southern hemisphere, which has its winter when the earth is farthest from the sun and its summer when nearest; though on the whole the southern hemisphere is coldest, for reasons which we shall see presently. But 11,000 years ago perihelion was in June instead of January, and it will be so again in 10,000 years more. And the difference of distances may be far greater than it has been for some thousands of years.

Glacial periods.—The eccentricity of our orbit has long been slowly decreasing, and is now near its minimum, and its variation affects the intensity of the seasons in the arctic and temperate zones to an amount enormously beyond what either astronomers or other philosophers* had believed possible, until the investigation of the subject by Mr. Croll, who has lately collected his previous papers into a book called 'Climate and Time.' Some parts of it are beyond the scope of this book, and have not the certainty of the strictly astro-

* It is a pity that this old-fashioned word has been allowed to acquire the meaning of either assumption or extravagant praise or ridicule, as the case may be, and that people are inventing instead various foreign and semi-English, or no-English-at-all substitutes for it.

nomical conclusions, which have not been seriously disputed since their first publication. The following may be taken as a summary of them.

According to the calculations of Le Verrier and others, it seems that the eccentricity has varied thus, taking only a portion of the longer table given by Mr. Croll, p. 320:—

The present eccentricity is		·0168	or a 60th
10,000 to 20,000 years ago it was		·0188	
40,000	minimum	·0109	nearly a 90th
80,000		·0400	
100,000		·0473	
150,000		·0332	
210,000	maximum	·0575	an 18th
250,000		·0258	
300,000		·0424	
350,000		·0195	
400,000		·0170	a 60th
550,000		·0166	
750,000	maximum	·0575	an 18th

There were some rather higher maxima long before, but that is immaterial.

Whenever in that long period of great eccentricity, from 80,000 to 300,000 years ago, or the earlier one 600,000 years before that, our hemisphere was at aphelion in winter, it was exposed to far greater cold than now. When the eccentricity was ·0575 we were above 97 million miles from the sun in winter (of which the square is 9409) against only 90 now. The mean winter heat of England is now about 39°. But that is only 39° above an arbitrary zero, which might be anything else, and by other thermometer scales is something else, for the others take the freezing-point of water for their zero. The real question is, how much hotter is it than if there were no sun? It has been concluded from experiments by Herschel and others that the zero

of space, or the heat in the absence of the sun, is 239° below our zero. It has also been concluded that the absolute zero in the absence of all heating influences is $-461°$; which makes the stars give us 222° of heat,* or not very much less than the heat due to the sun in winter—a somewhat strange result, which makes one suspect that the 239°, for the absence of the sun, is too high a temperature. However taking it to be right, our winter heat due to the sun when 90 million miles off is 278°. And it must vary inversely as the square of the distance. Therefore the heat of winter in aphelion with an eccentricity of ·0575 was $\frac{8100 \times 278°}{9409} = 239°$, or just our zero; but that coincidence is only accidental. If the zero of space is really lower, the difference between the winter heat of that period and the present would be still more than 39°, but that is quite enough.

You may ask however, why does not the increased summer heat in perihelion balance the increased cold of winter, and melt away the excess of ice so produced? At one time it was taken for granted that it must; but that was the mistake which underlay all the former conclusions on this subject. Though the direct heat of the sun is greater, it has to make its way to the ice through

* The mode of estimating this may be seen in 'Tyndall on Heat,' § 96, and he does not appear to consider it conclusive. It appears to me impossible. For star-light is practically nothing compared with sun-light; and yet some of the stars are estimated to have much greater intrinsic brightness than the sun, even as seen here. If the experiments on which these estimates of the absolute and sunless zeros of heat were conclusive, we should be forced to conclude further, that star-light is obstructed and turned into heat to an enormous extent in passing through space; but it certainly is not in the case of those stars; and these figures can only be regarded as approximations.

the vapour and fog which always rises from it, whether in a visible or invisible form. And though the air receives that heat, it then rises and carries it away in wind, and radiates it back into space; and so all that heat never reaches the ice, although in very clear weather the sun is sometimes hot enough to melt the pitch one side of a ship while the thermometer on the other side is below zero.

Secondly, ice is a fixed treasury of cold in a very different way from water. Water at 32° contains 143° more heat than the same weight of ice at 32°; for it takes as much heat to melt a pound of ice as will raise a pound of water 143°. This is called the *latent heat* of water. Colder ice of course takes still more heat to melt it. And the general temperature over a large mass of ice cannot rise until the whole is melted. Moreover, even if the same quantity of cold (as we may say) could fall in the form of water 143° or more below freezing, a vast deal of it would run away by mere gravity, and in currents of the sea, which would bring return currents of warm water; just as our temperature is now compounded of the heat directly received from the sun, and of the warm currents of air and water in these latitudes from the south-west, especially the Gulf stream. But when everything is covered with ice many feet and even miles thick, as it once was all over England, there can be no such interchange of warm and cold water, and the sun has to do the whole work of melting the ice through the fog, and has not time to do it in the summer. The same thing prevails now in high mountains all over the globe. It is colder there because the heat radiates

away faster into space through the thinner air, and more snow falls because the vapour raised by the sun is condensed by the cold mountain tops, and more falls in a year than the sun can melt, and so there is constant glaciation. The density of the air diminishes upwards so fast that half the weight of the atmosphere lies within 3½ miles above sea level, and there is no sensible atmosphere at 80 miles; and the density of the invisible vapour in the air decreases with it, which is the great obstructor of the passage of heat, and obstructs radiation of heat from the earth more than the reception of heat from the sun. But for the air thus retaining the heat every green thing on the earth would be killed by frost every fine night, as it would all radiate away, and they would be scorched by day. The clouds reflect the heat back again, and so cloudy nights in summer are the warmest.

A perihelionic summer, from equinox to equinox, is shorter than an aphelionic one, because the earth goes fastest at perihelion; but that does not signify, because the heat and the motion of the earth both vary inversely as the square of the distance, and so the quantity of summer heat received by each hemisphere is always the same; except that the earth receives on the whole rather more heat when the orbit is more eccentric, i.e., narrower; for then the average (not mean) distance for the whole year is evidently less, the minor axis of the ellipse being smaller while the major is constant. It is easily proved by mathematics that the total annual heat in any orbit varies inversely as its area. And as the area of an ellipse is 3·1416 × the product of the semi-axes, you see at once that the annual heat in an

orbit of given major axis varies inversely as the minor axis. In ellipses of small eccentricity the minor axis is to the major as 1 minus half the square of the eccentricity is to 1; and so you may calculate from the above table that the smallest minor axis is not a 700th less than the greatest; which would make no sensible difference in the total annual heat; but the difference between the greatest and least distance in a time of greatest eccentricity is a ninth, and it is that which so seriously affects the seasons and climates.

The glaciation also tends to increase itself so long as the necessary conditions last, by the accumulating ice depressing that part of the globe, or raising the earth's centre of gravity towards the glaciated pole, and therefore the water with it, which then gets frozen, and so extends the ice still farther, which again cuts off more heat in the next summer. And further still, the increased heat of the summer sun raises more water to fall in snow; and so glaciation goes on increasing until that winter leaves aphelion by the precession of the equinoxes.

Moreover, we shall see afterwards that the *trade-winds*, which blow pretty constantly from the north-east in the north hemisphere, and south-east in the southern one, are caused by the difference of heat at the poles and the equator; and that difference is of course greatest in the hemisphere which is most glaciated; consequently the trade-wind of that hemisphere carries some of the equatorial warm water over into the other, and in fact does so now, making the calm region north of the equator, and so tends to aggravate the difference of temperature between the two hemispheres still more;

for the great oceanic warm currents play a most important part in the warming of whatever lands they reach; while the returning cold currents from the polar regions always go underneath, and so produce scarcely any effect on the land. It is found that the water at the depth of only 600 yards at the equator is no more than 10° above freezing.*

All this is reversed in the warmer hemisphere, and may be reversed so much as to equalise the seasons there. The perihelion distance 210,000 years ago was only 87 millions of miles, of which the square is 7669, and substituting that for the former 9409 makes the winter temperature 293° above the zero of no sun, or 54° above our zero, which is only 6° below our mean summer heat. There would therefore be perpetual summer in one hemisphere whenever the other is in a state of maximum glaciation; even without taking into account the effects of the stronger trade-wind then sending over still more of the equatorial warm water, which we cannot calculate. For the last 60,000 years however nothing of this kind can have taken place, because the eccentricity has been too small; but even now the antarctic glaciation extends much farther than the arctic, though the antarctic summer is hotter beyond the limits of glaciation.†

* 'Climate and Time,' p. 119: and see further papers by Mr. Croll in the Ph. Journal, 1875.

† It cannot be due to this cause, but it is an agreeable fact, deducible from the published Greenwich tables of temperature, that our mean winter heat has increased about 1° in every 27 years in the last 105, the spring temperature rather more than half that amount, autumn rather less, and summer only a fifth or 0·2° in the 27. The rainfall of the London district had also decreased an inch in 60 years, but in the last 10 or 12 or 14 the average has slightly risen again. Many small rivers have vanished within living memory, and others are diminishing;

Mr. Croll has also made out that much greater changes of climate than had before been thought possible are due to the variation of the inclination of the earth's axis, which has diminished from the earliest days of astronomy about $\frac{1}{2}''$ a year. It is now $23^\circ\ 27'\ 44''$, and astronomers have calculated that its maximum was $24^\circ\ 36'$, and its minimum 22°, and that one maximum was about 11,000 years ago. So there would be some glacial epochs coinciding with a maximum, and others with a minimum inclination. If the axis were upright the poles would receive no heat at all, for the sun's rays would only graze them. On the other hand, if the poles lay in the ecliptic, they would alternately receive as much heat as possible. Therefore the greater the obliquity the more heat the polar regions receive. It has been calculated that they received an 18th more with the maximum obliquity than now, and therefore above an 18th less than now with the minimum obliquity. Each 18th is equivalent to 15° of heat by the method of calculation at p. 53. So here is another range of 30° of heat, to be compounded with the former according to circumstances, at some periods increasing the effects of glaciation and submergence of one hemisphere, and at others tending still more to make the polar regions temperate, and even warm, all the year round.

All this concurs with geological evidence from various parts of the world; in the evidently successive submergences of a great part, if not the whole, of Europe

no doubt partly from artificial drainage, and partly from the continued cutting down of woods, which is quite ascertained now to diminish the rain of the neighbourhood.

and Asia and North America to different depths below the sea, and emergences again, and traces of the grinding of glaciers over rocks now high; and in their successive beds of coal, which is composed of trees and plants of interglacial ages, and such as could only grow in a warm equable climate; and divers other evidences, for which you must consult Mr. Croll's book.

He has also deduced, with all but astronomical certainty—a very different thing from mere geological estimates—a limit to the time during which the three northern continents can have had any existence at all for vital purposes; and that limit is very far within the millions of years which some recent theorists have been playing with as if they had any number of such ages at their command. For we see that about 210,000 years ago there was a period of maximum glaciation here, besides the minor ones in much less distant times, in which a great part of this hemisphere was covered with perpetual ice for thousands of years together, and parts farther south, which may have escaped that, were kept down under the sea by the weight of that same ice and overflow of water from the other hemisphere then unfrozen.

These are tremendous consequences to follow, as they do, from such a remote and small cause as the disturbance of the inclination of the earth's axis and of the eccentricity of the orbit by the attractions of the other planets; of which neither the largest nor the nearest occupies a space in the sky = a 2500th of the moon's, or larger than a pea at the distance of 125 feet. Even that gives much too large an idea of their power of disturbance, for that does not vary inversely as the

square of the distance, as attraction does, and as apparent size does, but as the cube of the distance, as we shall see afterwards. And therefore, although Jupiter is about 25,000 times as heavy as the moon, his disturbing force on the earth is only a 320,000th of the moon's, and Venus's about the same; and the effects of the other planets are much less.* Yet these disturbances, you see, produce effects on the earth compared with which the tides are as nothing. For they destroy a large part of the earth for the time as completely as if it were burnt up, or crushed under another planet; and at other times they bring constant summer into the regions previously buried miles deep under perpetual ice.

Length of day and night.—We have only hitherto spoken of summer and winter as the extremes of heat and cold, and without reference to the earth's rotation. But we know that summer is not only the time of warmth, but also of longest days; and to explain that we must consider the earth's rotation as well as revolution. Take a terrestrial globe and elevate the north pole $23\frac{1}{2}°$ above the wooden horizon, which we may take to represent the boundary of light and darkness, assuming the sun to stand right above it; for of course there is no such thing really as 'above and below' in the heavens, and we only use these terms for convenience. Then as you spin the globe round for its daily rotation, you will see that nothing within the *arctic*

* The moon does also produce other disturbances of the earth, besides the tides which counteract themselves daily, instead of accumulating like the climatic disturbances. She sways the earth backwards and forwards 5790 miles every fortnight, besides being the chief cause of the precession of the equinoxes, as we shall see presently.

circle, $23\frac{1}{2}°$ from the north pole, ever goes below the horizon or into darkness; that is, the sun never goes below the horizon of the people within that circle in the northern midsummer. At the same time nothing within the *antarctic circle*, $23\frac{1}{2}°$ from the south pole, comes into the light at all, and so those people have no daylight in the middle of their winter. The converse of all this evidently takes place at the opposite time of the year. But half-way between those times, when the earth is either east or west of the sun, the two poles are equidistant from the sun; and so the light received by the two hemispheres is equal. And to measure the length of days at those times you must lay the poles level with the horizon or boundary of light, and you will see that every part of the globe is just as long above as below it, or the days and nights are equal.

Therefore those times are called the *equinoxes*, which occur on March 21 and September 23. Midsummer and midwinter are called the *solstices*, because the sun then stays at the same distance from the equator for a few days, as you may see by looking at the ecliptic on a globe where it is farthest from the equator; and the days remain of the same length for a short time before they begin slowly to get shorter or longer again. The nearer you go to the poles the greater is the difference of days and night at all times except the equinoxes. Even in the north of England the difference is visibly greater than in the south.

The reason why it is so cold where the sun never sets, is that his rays generally fall very obliquely on the polar regions, as you may see, considering the sun right above the globe when the pole is elevated $23\frac{1}{2}°$ above

the wooden horizon. It is true that he is higher above the horizon of any place on the arctic circle at noon in midsummer, than he is here in winter, viz., 47° (twice $23\frac{1}{2}$) against our 28° (lat. $51\frac{1}{2}° - 23\frac{1}{2}°$); and in fact the sun is sometimes very hot there. But in the shade it is always very cold, from the absorption of all the heat by the ice and snow; which may be called the radiation of cold, as cold bodies absorb heat from hotter ones in every direction.

The polar regions in short are now in a glacial condition, and so are the tops of mountains where more snow falls in winter than is thawed in summer. Professor Tyndall's explanation of that is, that the air, and the vapour in the air, decreases rapidly in density upwards, and the vapour absorbing the radiated heat of the earth leaves less and less of such heat the higher you go, though the sun's rays are actually hotter, and sometimes hot enough to blister your skin in a generally freezing temperature. Vapour stops radiated heat much more than direct sunheat. That is why one gets so hot walking in a damp day.

At the equinoxes the sun appears on the equator, which means in astronomy not merely a circle round the earth equidistant from the poles, but the plane of that circle extended to the heaven. And as the sun is always in the ecliptic, the equinoxes are the places where the equator and ecliptic cross each other. For all these purposes we may properly talk of the earth as turning on a fixed axis, and the sun moving round the earth in the ecliptic. For their relative motions are the same as if they did so; and if there were no other bodies in the universe to measure by, no human

being could ever have found out that the earth does not stand still with the sun revolving round it; or that the sun is bigger than the earth.

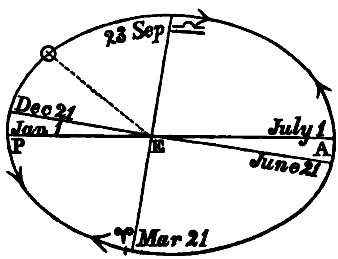
The equator is necessarily as much inclined to the ecliptic as the poles of the earth are to that line perpendicular to the ecliptic which is called the *poles of the ecliptic*, whether it is in the sun or the earth. Therefore two 'small' circles, each 23° 28′ from the equator, are the boundaries of the sun's journey to the north and the south of the equator. They are called the *tropics*, which means the turning places of the sun; the northern one is the tropic of Cancer, which the sun touches at our midsummer, and the southern is called the tropic of Capricorn, which the sun reaches in our winter and the southern midsummer. The band between the tropics is the *torrid zone*, the two arctic circles contain the *frigid zones*, and the spaces between are the *temperate zones*. They cover respectively ·4, ·08, and ·52 of the earth's surface.

But you may ask, why should the torrid zone be always hot, since the sun is 47° away from each tropic when he is at the other, while he is almost directly overhead to parts of each temperate zone when he is at the tropic nearest to it? The reason is, that places within the torrid zone get a greater quantity of sunshine nearly or quite direct in the whole year than any places can outside of it; and the heat is accumulated, or as it were bottled up in the earth, and stays there after the sun has left that place or latitude. The heat received anywhere depends on the directness of the sun's rays, or its apparent verticality overhead; for a square foot or a square mile of surface evidently catches more or less rays from a fire or the sun according as it faces them

directly or is turned obliquely towards them. Moreover, when the sun's rays come from a low elevation above the horizon they have to pass through a much greater length of air than when they come nearly vertical. The air strips the rays of some of their heat, though not actually in proportion to the length of air passed through. Every place within the tropics has the sun directly over it not only once, but twice a year, and has in fact two summers, one as the sun is going from the equator to the nearest tropic, and another as he returns over the same latitude towards the equator, where the two equinoxes are the proper summers, the sun being then vertical there. In those regions therefore a quantity of heat is accumulated which no place beyond the torrid zone can get. Still it is sometimes hotter in the low latitudes of the temperate zones than it is at other times within the torrid zone. More heat is gained in the long sunny days than radiates away in their short nights, and the excess accumulates and makes July and August hotter than June and May.

The equinoctial points, where the planes of the equator and ecliptic cross each other, are of great importance in astronomy, because nearly all the celestial measures are reckoned from the point of the vernal equinox, which is called the first point of Aries ♈; the autumnal one being the first point of Libra ♎. Aries and Libra are also the names of two clusters of stars or *constellations,* which were imagined by the ancients to represent a ram and a pair of scales; and they are still kept as the names of two of the divisions of the ecliptic into twelve parts called *signs of the zodiac,* which are these:—Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋

(of which the first point is the summer solstice, from which the northern tropic is named), Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricorn ♑ (where the winter solstice is), Aquarius ♒, and Pisces ♓. But they are now seldom used in astronomical books. About 2200 years ago the sun used to enter the constellation Aries when he also entered the sign Aries or the equinoctial point; and in the earliest ages of astronomy the equinox was in Taurus. But now the sign ♈ has left the constellation Aries, for the reason you will see presently, and is in Pisces.



As contradictory statements about the movement of the seasons in long periods have appeared in other books, I add this figure, with some exaggeration for distinctness, both of the eccentricity of the orbit and of the distance of perihelion P and aphelion A from December 21 and June 21. As we are only concerned with the relative motions, it is better to represent the sun ☉ as going round the earth E, and so appearing (as he does) to enter ♈ in March and ♎ in September. The sun and the apsides PA both move in the direction of the arrows at PA; but

the equinoxes go the other way among the stars, as marked by the arrows at ♈ and ♎.

Trade-Winds.—The heat of the torrid zone and its velocity of rotation produce these winds, which blow constantly in the same directions in the same latitudes on the great oceans; though not so constantly on land, on account of variations in heat and other causes of disturbance. The heat expands the air and makes it rise from the equatorial regions; and then the denser air from cooler latitudes comes in, and would make a constant north wind (in this hemisphere) if the earth were either stationary or cylindrical. For if the earth had no rotation the air would have no east or west motion; and if it were a cylinder all the air would be carried round with it from west to east with the same velocity, and would be no more felt as a wind than the air you carry with you in a railway carriage, though it moves as fast as a very high wind. But the earth's surface moves 1040 miles an hour at the equator, 900 at latitude 30°, only 520 at latitude 60°, and at the poles its velocity from rotation sinks into nothing.

Therefore while the air comes south (in our hemisphere) it is always coming to a place which moves faster eastwards than the place it came from; and so the north wind becomes north-east; just as a weathercock would point N.E. in a north wind if you carried one with you running east. This is the principal trade-wind, blowing from N.E. in the northern hemisphere and from S.E. in the southern, up to about latitude 30°. Near the equator its eastern character is lost, because there is no material increase of velocity in the earth as you get very near the equator. Also the north and

south winds meet there, and make a calm: but the line of greatest heat and calm is a little north of the equator, partly from the cause stated at p. 57, and also because the much greater quantity of land in the northern hemisphere retains heat more than water which can move away.

The air which rose from the equator must go somewhere, and it goes in an upper current towards the poles, and begins to fall again when it gets cool, to fill up the space left by the air coming to the equator. And as that air from the equator started for the north with an eastward velocity of 1040 miles an hour, and comes down again on latitudes which move much slower, it is felt there as a S.W. wind in this hemisphere, and N.W. in the other. This secondary or *anti* trade-wind prevails from about 30° to 60° latitude at sea, and makes ships sail from North America to England nearly twice as fast as from England to America.

It is a disputed question whether the great sea currents, such as the Gulf stream, are produced in the same way, by the difference in weight of warm and cold water causing a flow of warm surface water from the equator towards the poles, and a return of the cold water underneath; or by the action of the prevailing winds upon the water, modified by its impact on the various shores which turn the currents aside. Dr. Carpenter is the chief advocate of the gravitation theory, and Mr. Croll maintains the wind one, and appears to me to have decidedly the best of it, but the subject is too large to discuss here.*

* See also Proctor's 'Light Science,' vol. ii., which however does not enter into the controversy between wind and water.

The Gulf stream is said to be equivalent to a river of warm water 100 miles wide and 500 feet deep, flowing 4 miles an hour and conveying as much heat from the torrid to the frigid zone as that zone receives from the sun in the whole year.

PRECESSION OF THE EQUINOXES, AND LENGTH OF THE YEAR.

I said at p. 44 that the earth goes round the sun, so as to see the same star again in a line with him, or in *conjunction*, in about 365¼ mean solar days. The exact time is 365·2563 days (omitting further decimals), and that is called a *sidereal* year. But the important thing for all ordinary purposes is the year of seasons, called the *tropical* or *equinoctial* year: and that is a little shorter than the sidereal for this reason. The equinoctial points ♈ and ♎ recede among the stars. If that imaginary line where the plane of the equator cuts the plane of the ecliptic points this year to any two given stars in the east and west, next year it will point 50″·1 behind them; so that the sun will reach the spring equinox, or cross the equator from south to north 20m. 20s. before he comes again into conjunction with the same star as before. Consequently, if we reckoned by sidereal years, the seasons would get sensibly wrong in no very long time. This is called the *precession of the equinoxes*, because it makes them precede their sidereal time. It was discovered by Hipparchus about 150 B.C.

The length of the equinoctial or tropical year is now settled by astronomers—at least by the English ones, to

be 365·242216 mean solar days, or 365d. 5h. 48m. 47½s.; and the French measure is practically the same. It is reckoned from the time when a mean sun going at the average speed determined from the longest experience would pass through the mean ♈; for that also has its variations, and the motion of 50″·1 a year is not quite uniform in each year, as you will see presently. But first let us try to realise what kind of motion the earth goes through to produce this effect.

Get a common celestial globe, and set the axis of rotation upright, and consider the wooden horizon to represent the ecliptic or plane of the sun's orbit. Then the axis of the globe will be the poles of the ecliptic, though they are not so on the globe; but we want the poles of the ecliptic as marked on the globe to represent the poles of the equator or of the earth for our present purpose. Now let our new north pole of the earth lean towards the north side of the room; and you may consider the sun as going round the earth from right to left, or west to east through south. If there were no precession the pole would always point the same way, and the sun would always be among the same stars at the same seasons. But the poles of the earth twist slowly round the poles of the ecliptic westwards, or the opposite way to the sun, keeping about 23½° from them, and going quite round in 25,868 years. Turning our globe slowly round its upright axis from left to right, you will see the equinoctial points recede along the wooden ecliptic: which makes the equinoctial year those 20m. 20s. shorter than the sidereal.

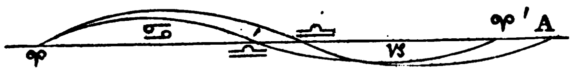
The pole star, or the equinoctial stars, of any epoch are thus an index to it in the great cycle of precession;

a Draconis, now 25° off the pole, was only 3° 42′ off when the Great Pyramid probably was built, about 2170 B.C., a few centuries after the Flood. Its entrance passage is 3° 42′ inclined to the earth's axis, and looks due north, or lies exactly in a plane through the earth's axis, with the wonderfully small error of only 5′, or 13 inches in the base length of 761 feet, and that in times when they had no telescopes. Therefore that pole star then looked straight down that long narrow passage exactly at midnight on the shortest day, and at its lower transit every day. And at the same time the south meridian was crossed by the Pleiades in ♉, where the equinoctial point was then, and which are associated with the beginning of the year by traditions and customs all over the world, including the well-known eastern worship of the bull.

Though the precession of the equinoxes was discovered by Hipparchus, the cause of it was first made out by Newton to be the difference between the sun's attraction on the nearer and the farther parts of the equatorial protuberance round the earth, which is equivalent to a ring of matter laid round the globe for this purpose. At all times except the equinoxes the nearer half of that ring is either mostly above the ecliptic and the farther half mostly below it, or *vice versâ*, and in either case that difference of attraction acts as follows. Let us take the time (our winter) when the half of the equator above the ecliptic is nearest to the sun. The sun's attraction being always more on the nearer half than on the earth's centre, and more on the centre than on the farther half of the equatorial ring, it is equivalent to a force which pulls the nearer half as it were with a

string to the sun, and therefore a little downwards towards the ecliptic, and also pushes away the farther half as it were with a long stick, which also tends to push that upwards towards the ecliptic. Then why has not the equator been long ago pulled into the plane of the ecliptic? Because of the earth's rotation. Attraction acts on each particle separately, and for each particle the ring is merely an orbit in which it travels round the earth's centre. We must therefore consider how each particle will behave under the two forces, the original one of rotation with the earth, and this extra force of attraction always urging it downwards or upwards towards the ecliptic. We need only consider it on one side, say the upper, as the tendency is the same wherever it is.

It will be easier to understand if we suppose the path of this particle round the earth (that is the equator) and the ecliptic, to be opened out into a flat picture thus,



the ecliptic being the straight line, and ♈ ♋ ♎ ♑ A the course a particle would take if there were no disturbance. Suppose a wind blowing down on the particle while travelling from ♈ to upper ♋, it will evidently depress its course a little into the lower curve; and as it goes on from ♋ towards ♎ the wind will still depress its course, and bring it down to the ecliptic at ♎′. But though the new course lies within the old one, it may still come down at ♎′ at the same angle as it would have done at ♎, only it reaches the ecliptic sooner, or ♎ recedes. From ♎′ through ♑ the same kind

of action takes place; and so instead of reaching the ecliptic at A, which represents the original ♈ opened out, undisturbed by the force urging it to the ecliptic, it will reach the ecliptic at ♈′, or ♈ has receded to ♈′; but the inclination of the path, i.e. of the equator, to the ecliptic, has not been altered at the places where they cross each other, though the equator is nearer the ecliptic at every moment than it would have been if there were no force tending to the ecliptic. For what happens to one particle of the equatorial ring happens to them all. This also may be shown by the instrument called the gyroscope mentioned at p. 42. The wheel is set spinning in an oblique position, and a small weight is hung near the bottom of the axis, which would pull the axis upright in a moment if the wheel were still, but when the wheel is spinning it makes this polar axis twist slowly backwards or opposite to the direction of rotation.

For the purpose of explaining the cause of precession I have only mentioned yet the sun's attraction; but in fact the moon contributes to it in the same way, and even more; for she also moves nearly in the ecliptic, and attracts the front of the ring or the part which is nearest to her more than the middle, and the middle more than the back parts which are farthest off. Moreover, as the effect of the sun and moon on precession is all due to the *difference* of attraction on near and far parts of the earth's protuberance, the moon really does more towards it than the sun, though her attraction upon the whole earth is very little compared with the sun's, who is enormously larger and heavier; but then she is very much nearer; and the difference between

the back and front of the earth is a 30th of the moon's distance, but not quite a 12,000th of the sun's, and their differential force is inversely as the cubes of their distances, which you must take for granted for the present. The result is, that giving the moon the benefit of her nearness, and the sun the benefit of his weight, the moon does above twice as much as the sun in producing the precession; as we shall see afterwards that she also does in producing the tides by the same difference of attraction on the opposite sides of the earth.

The lunar part of the precession varies like the solar, according as the moon is near or far from the equinoctial points; and so it may be least when the solar precession is greatest; or they may both be at their maximum or minimum together. The $50''\cdot1$ is the average or mean precession in a year.

Nutation.—There is yet another irregularity in the lunar precession. In consequence of the moon being sometimes a little above, and sometimes below the ecliptic, she does not pull quite on a level with the sun, and so produces a sort of nodding of the pole from its average motion in a circle round the pole of the ecliptic; and that is called *nutation*. The poles of the earth, or of the equator, go like a man walking between the rails round a race course, but with a wavy motion from one side to the other, instead of walking along the middle. But the word 'nutation' is used to comprehend all the variations of precession, both forwards and sideways. $9''\cdot2$ is the extent of the nutation on each side of the middle or average course of the pole in its circle of $23^\circ\ 28'$ radius round the pole of the ecliptic; and the length of each wave, or rather half-wave, from one

crossing of the middle of the course to another, is 3′ 10″, corresponding to 9·3 years, or half the time of one revolution of the moon's nodes, or places where she crosses the ecliptic. You must understand that this 3′ 10″ is not measured round the pole of the ecliptic as a centre (in which case it would be $9{\cdot}2 \times 50''{\cdot}1$), but as an arc of the great sphere of the heavens, with the earth's centre for its centre, as all the celestial measures are.

The amount of the nutation, or disturbance of the earth's axis by the moon when acting in a different direction from the sun, is one of the means used for calculating her power of attraction, or mass, compared with the sun's. The protuberance of the earth disturbs the moon in return, as will he explained when we come to the greater oblateness of the planet Jupiter, which disturbs his moons still more.

Before we leave precession, I should tell you of a curious use that has been made of it, towards settling the question whether the earth is a thin shell of rocks full of melted lava or other fluid inside, as some persons supposed. The late Mr. Hopkins of Cambridge calculated that the precession would be greater than it is if the earth is not solid for about 1000 miles deep. For the sun and moon would twist the axis of the earth rather more if the protuberance under the equator were fluid; which may be roughly illustrated thus:—A pendulum with a bob made of a glass globe filled with quicksilver swings rather faster than a pendulum similar and equal in all respects except in the bob being solid. The reason is that the hollow globe does not stick to the mercury and hold it fast, but slides round it as it turns a little in swinging, and so the

mercury does not turn with it; whereas the whole of a solid bob has to be turned as well as swung at every vibration, which uses up more of the force of gravity, on which the quickness of the pendulum depends. If the earth were not hot within, all subterraneous water and all the bottom of the sea would be frozen, and the earth itself below a very little depth.

Not only do the points of crossing of the equator and ecliptic recede 50″ a year along the ecliptic, but the *obliquity of the ecliptic* itself, or its inclination to the equator, decreases about half a second a year (p. 59). It was 23° 51′ in 230 B.C. The reason is that the whole ecliptic or plane of the earth's orbit is slowly tilted by the attractions of the other planets. Moreover, as it neither turns on the line of equinoxes, as a hinge or axis through the sun, nor directly across it, it makes the annual precession a little less than it would be otherwise, and also lets it increase a *very* little, so that the tropical or equinoctial year is 12 seconds shorter than it was 2000 years ago, though the absolute time of the earth's revolution round the sun, or the sidereal year, has not altered.

Sidereal and Solar Days.—We must now consider the different measures of a day. And first we may remark that the day of astronomers begins at the noon after the midnight when our common day begins, and has no A.M. or P.M., but simply 24 hours. Thus 11 A.M., 1 January 1876, was 23 o'clock 31 December 1875, in astronomical almanacs. But there is also a time called *sidereal*, which is still more different. The sidereal day here begins and ends when the equinoctial point ♈ crosses the meridian of Greenwich. If the sun

is on the meridian at the same time (as he is only at the vernal equinox) he will not have quite got there again by the time ♈ is there again, or at the end of that sidereal day; because the earth has meanwhile moved on a day's journey in her orbit and passed the sun a little, and so has to turn a little more than quite round for Greenwich to face the sun again, since she rotates in the same direction as she revolves round the sun, from west to east, like a small wheel of 8 teeth rolling round a very large one of 2922. For that small wheel would turn 365¼ times relatively to the large one in one revolution round it, but 366¼ times absolutely. So there is one more sidereal than solar days in a year. More exactly, a sidereal day is ·9973 of a mean solar day, or 3(solar)m. 55·11s. shorter; or a solar day is 1·002738 of a sidereal one; and the hours and minutes are in the same ratio.

A sidereal day then is practically the time of one absolute revolution of the earth, or the time between two transits of the same star. For the precession of the equinoxes makes no sensible difference in a day, and it is practically the same thing whether we call a sidereal day the time between two transits of ♈ or two transits of the same star. Not only is the daily precession 366 times less than the annual, but the motion of ♈ through 50″·1 only makes a difference of 3⅓ seconds in the time of the clock, or in the arrival of ♈ at the meridian at the end of a year, though it makes 20m. 20s. difference in the time of the sun (or earth) reaching ♈ again. There is however a little inconsistency in the use of the word 'sidereal' for days and years; for a sidereal year measured by the stars

is contrasted with an equinoctial year measured from ♈, while a sidereal day is measured from ♈ and not the stars, and contrasted with a solar day. The sidereal day never begins at 12 o'clock, except at noon on the 21st of March, and at midnight on the 23rd of September.

Equation of time.—We have yet to inquire what a solar day really is. I called the equinoctial year 365·242216 *mean* solar days. A true sun-dial solar day is the time between two transits of the sun over the meridian. But such days are by no means of equal length, and so far different, that in November the true solar time is a quarter of an hour before mean time (which is clock time), and in February a quarter of an hour behind it. That is the reason why the afternoon light appears to last so much longer after Christmas than at the corresponding time before Christmas. The causes of the inequality of solar days are the unequal velocity of the sun in his elliptical (apparent) orbit, and still more, the unequal motion of the sun in the direction parallel to the equator, in consequence of the obliquity of the ecliptic to the equator. The mean solar day then is the average of all these variable solar days. And the *equation of time* is the difference between the time of day by the clock of mean time and the time by the sun-dial. It is the column headed in the almanacs 'Clock before sun,' or 'Clock after sun.' It is practically the same every fourth year, like the time of sunrise and sunset, and would be quite the same if the year were exactly 365¼ days, as leap year would restore it then exactly.

CHAPTER II.

THE SUN.

In considering 'the solar system,' which means the sun and all the planets and their moons or satellites, it is important to get a clear idea of the size and weight of the sun, as it is his attraction that keeps them all in order, moving in their proper times and distances, besides his other equally important business of sending light and heat to them—if they all have inhabitants to use it. According to the latest estimate of dimensions of all the solar system (of which I shall have to say more presently) the sun's diameter is 857,700 miles, or 108·23 times the earth's equatorial diameter, but 108·32 of the diameter of a sphere of equal bulk. But that gives you no idea that he is above a million and a quarter times as big as the earth; which he is. If you take two bricks and lay them lengthwise, and then put two more alongside, and then put four more on the top of those four, you will have made a thing of the same shape as a brick, but twice as long, twice as broad, and twice as high, and therefore eight times as big altogether; if you treated three bricks in the same way you would make one twenty-seven times as big. And the same is true whatever the shape is; for a body of any shape

may be made up of an infinite number of infinitely small cubes; and if each of them is doubled or trebled in every dimension, the body will be doubled or trebled in every dimension, and will therefore be increased eightfold or twenty-sevenfold in bulk. This is expressed by saying that the bulk, or volume, or solid content, and (if the densities are equal) the mass, varies as the cube of the diameter. Therefore the sun is $108 \cdot 32^3$ or 1,271,470 times as large as the earth.

The sun's distance from the earth varies from 90,436,000 to 93,564,000 miles. The mean between the greatest and least is called the *mean distance;* and that is now considered to be 92,000,000 miles, though you will find it in older books called 95 millions, because it was believed to be so until lately. You will hear more of that correction as we go on, and also how the diameter and distance of the sun are both measured. The distance is also 23,213 times the earth's equatorial radius, and is certainly 108·25 times the sun's diameter whether that and the distance are yet correctly measured or not. (The agreement of this figure with the proportion of the sun's diameter to the earth's is one of those remarkable coincidences, of which we shall meet with several, which are quite accidental, though so close as to suggest the erroneous idea that there must be some physical reason for them.) For the sun's apparent diameter can be measured easily by mere observation of the angle between the rays of light or two straight lines coming from his opposite sides to a telescope; and that angle (see p. 9) is simply his diameter divided by his distance, whether they be as great

as we know them to be or as small as the ancients fancied. The real difficulty is in measuring the proportion of the earth's diameter either to the diameter or the distance of the sun. We shall see afterwards how that is done.

As nearly all distances in the solar system are measured by millions, and nobody ever counted a million of anything, it is worth while to stop a little to understand what it is, by the help of a few specimens. A million days are 2730 years; so there have been not much more than two million days since the creation of Adam, according to the Hebrew chronology, and rather more than one million since the time of Solomon. A railway train going thirty miles an hour and never stopping would take nearly four years to go a million miles, and eleven years to go once round the sun, and three hundred and sixty years to go from here to the sun. A cannon ball, or the sound of it, which goes about the same pace, 1130 feet in a second, would take nearly 15 years to reach the sun, supposing the ball to go on with its initial velocity. If you had a million shillings to count one by one, and did it as fast as you could for ten hours a day, it would take a fortnight; and the million shillings would weigh nearly six tons, and would reach nearly 15 miles if laid in a row. A million is a thousand thousands, a milliard a thousand millions, and a billion is a million millions; a trillion is a million billions, and so on.

The sun's enormous size will be better apprehended from the fact that he is 6 times larger than a globe which would contain the moon's orbit, his radius being nearly twice the radius of that orbit. And yet three of

the half-a-dozen stars which alone astronomers have been able to measure, or rather to weigh (as you will see hereafter), are much larger: one of them 28, and another probably 80 times as heavy as the sun. He is quite round, not spheroidal like the earth, because he rotates too slowly for centrifugal force to produce any sensible oblateness.

Although the sun is above a million times as big as the earth, he is only 322,700 times as heavy, and is consequently made of much lighter materials. Or more probably we do not see the real sun at all, but only his luminous envelope, which may be much larger than the sun himself. Since the attraction of a globe on any part of itself varies as the radius up to that point (p. 35), it seems very unlikely that the density of the real solid sun is less than the earth's. Measuring by his apparent size, his density or specific gravity is only ·254 of the earth's, or not much greater than water, and rather less than if he were made of coal. You must not suppose that we can measure the sun's density except through his size and his weight, which has to be calculated from his effect upon the earth's motion, as I shall show you towards the end of the book. The density of anything is in direct proportion to its weight and in inverse proportion to its bulk; and as we can calculate that the sun is only a quarter as heavy as he ought to be according to his size in proportion to the earth, we know that his average density is only a quarter of the earth's. The way to find the density or specific gravity of anything heavier than water, which is always taken for the unit, is to weigh it first out of the water and then hanging in water by a string as thin as possible, and the

specific gravity is the weight in air divided by the difference of the two weights.

In the table at the end of the book I have given the specific gravities of the sun and planets, assuming the earth's to be 5·5, which is the greatest of them all except Mercury, and very much greater than all but Mercury and Venus. Saturn is as light as deal and the light woods (not cork as Sir J. Herschel says, for that is only ·24); and the other three great planets and the sun are about the same as the very heavy woods and coal. But we shall see that there is reason to doubt whether we see the real bodies of Jupiter and Saturn, and not merely a dense envelope of clouds. Mars and the moon are about as heavy as diamonds and heavier than stones, which are generally 2·7, but much lighter (as the earth is too) than any of the common metals except aluminium, which is only 2·25 or the same as glass. All these densities, remember, are only measured through the earth's, which depends on the experiments described in the first chapter.

From the diameter and mass of the sun, compared with those of the earth, we can calculate their comparative attractions and the force of gravity at their surfaces. For by the law of gravitation, that is directly as the mass and inversely as the square of the distance from the centre. Therefore, since the sun's radius is 108·32 times the earth's,

$$\frac{\text{gravity on the sun's surface}}{\text{gravity on the earth's surface}} = \frac{322700}{108{\cdot}32^2} = 27{\cdot}5\,;$$

or a man on the sun would feel as heavy as if he had nearly 27 others laid upon him, and would be squeezed flat by his own weight.

New Dimensions of the Solar System.—The proportionate dimensions of the solar system beyond the moon, i.e. the distances, diameters, bulks, and masses of the sun and planets and their moons, are comparatively easy to discover, while the absolute dimensions are very difficult. The mode of finding the proportionate distances belongs to a later part of the book. The diameters follow immediately from the distances, because the apparent diameter, or the angle between two rays of light coming from opposite edges of a planet to a telescope, is simply the real diameter divided by the distance, and that angle can be measured easily. The bulks are $\cdot 5236$ (or $\frac{\pi}{6}$) $\times$ diameter^3, allowing for the spheroidicity if any (p. 19). The masses also depend on the cubes of the distances for another reason. It is a necessary result of the law of gravity, as we shall prove afterwards, that the mass of a sun or planet $=$ $4\pi^2$ (or $39{\cdot}48$) $\times$ the distance^3 of any body revolving round it $\div$ the (time of revolution)^2. Therefore as between any two planets, the cubes of their distances must be in the same proportion as the squares of their periods; and so the proportionate distances of them all are known, even without the trouble of observing anything more than their periods; though the proportionate distances had been found before this law was discovered by Kepler as a matter of observation, and afterwards proved to be necessary by Newton in the form just now stated. And that enables us to go a step farther, and to lay down the proportionate weights of the sun, and of all the planets which have moons. For you will easily see that it follows that the mass of Jupiter is to that of the sun as the distance^3 of any

moon of Jupiter $\div$ its period^2 is to our distance from the sun $\div$ our year^2; and as those distances are proportionately known, and the periods, we have at once the proportion between the mass of Jupiter and the sun. The proportionate masses of the planets without moons are determined by a much more complicated calculation from their disturbances of each other and of comets passing near them, at distances which are observable in proportion to all the other distances, though not absolutely.

The moment we can find one real distance or diameter therefore, we get the scale for measuring the whole solar system. Our moon is so near that we can find its distance separately, but also too near to enable us to determine the greater distances of the sun and planets by it — at least not so accurately as by some other methods. For nearly a century, until about 20 years ago, the sun's distance, which is naturally taken as the standard for them all, was considered to be 95 million miles; and so it still stands, even in Herschel's 'Outlines of Astronomy,' uncorrected, save by a note at the end of the several latest editions intimating that that is probably nearly 4 millions too much. We shall see afterwards how this excess was first suspected by Hansen from a very small disturbance of the moon not quite agreeing with the calculation founded on the larger sun's distance. That suspicion was soon after confirmed by some experiments on the velocity of light, which was found to be about 185,000 miles per second, whereas it ought to have been 192,000 according to the supposed distances of the sun and Jupiter; for the eclipses of Jupiter's moons come 16½ minutes sooner when he

is nearest to us than when he is farthest off; in other words, light takes that time to travel over the diameter of the earth's orbit. And then came some observations of Mars, which will be described afterwards, and some other things, all tending to the conclusion, that the sun's distance lay between 91,400,000 and 92,000,000 miles; and about the year 1864 astronomers came to adopt 91,400,000, or about a 24th less than before.

Since the cubes of the distances of all the bodies (the periods being fixed) must vary with the mass of the 'primary' round which they go, it followed that when the distances were reduced a 24th, the masses had to be reduced nearly an 8th; as you may see if you compare 24^3 and 23^3; or more shortly by the approximate rule in the note at p. 36. And the diameters of the sun and planets must vary as their distances, as I said just now, and the bulk as the cube of the distances, like the masses, though for a different reason. So we may say shortly, that the discovery of the error in the sun's distance derived from the transit of Venus in 1769 reduced all the linear dimensions of the solar system a 24th, and the volume and the weight of all the members (beyond the moon) nearly an 8th.

Here I stop for a moment to remark how much better the old, and the mathematical, habit of using vulgar fractions in small figures is than the modern vulgar habit of *per-centing* everything. It is very easy to remember, when you have once learnt it (which few people seem to do), that a reduction of a 24th corresponds to an increase of a 23rd, a reduction of one third to an increase of one half, and so on; or that any two numbers, such as 2 and 3, which differ by one half

of the smaller number differ by one third of the larger. But no such rule can be expressed in decimals. Some people nowadays will not even talk of half or two thirds of anything, but of '50 per cent.,' and '66 and two thirds per cent.;' and sometimes they talk of a thing costing 100 per cent. *less*, as if it were the converse of costing 100 per cent. *more*, instead of being nonsense; for they do not mean that it costs nothing, but half as much. If they would condescend to use vulgar fractions whenever they can be expressed in small figures, and especially when the numerator is 1, they would save a great many words and make fewer mistakes. In some cases decimals are best.

Latterly however, astronomers had come to think that they had reduced the distance rather too much; but no new figures had been definitely adopted, because they were waiting for the transit of Venus on the 8th of December 1874, which it was hoped would settle the question as exactly as it is capable of being settled, subject to another, but less favourable, transit in 1882, the last there will be until 2004. But the process involves a comparison of numerous pairs of observations taken as far apart on the globe as possible, and a great deal of calculation from them; and up to the time when this is written only an approximate result has been announced, that the sun's mean distance is about 92 millions of miles, instead of the 91,400,000 at which it stood for a few years.

I should tell you however, that astronomers do not speak of the sun's distance in miles: they always describe it by what they call his *parallax;* of which it is enough to say at present that it is the earth's apparent

equatorial radius, 3963·3 miles (not diameter, remember), as it would be seen from the sun's centre; or it is the earth's radius divided by the sun's distance, which you may easily calculate, from the figures above, is ·0000043, or 1 ÷ 23213, as given at p. 80; and by the rule for the numerical value of angles at p. 9 you will find that that represents the angle 8″·88, which is the received parallax at present, though it may be slightly modified when all the transit observations are worked out. The old parallax derived from the transit of 1769 was 8″·56, and that adopted in 1864 was 8″·94, which was used in the previous editions of this book. I shall give some account of the transit operations afterwards, and of the other methods for ascertaining the parallax of the sun and moon and planets, and also of the very few stars within a measurable distance.

Sun spots.—Sir W. Herschel's theory is still accepted, that the solid sun is a smaller body inside the luminous shell which we see, and which is called the *photosphere*, or light-giving sphere of the sun. The spots which sometimes appear are holes in the photosphere, but whether reaching to the bottom of it is by no means certain: it is now thought not. Their depth has been estimated at 30,000 miles; and they have enabled us to measure the time of the sun's absolute or sidereal rotation on his own axis, in 25d. 7h. 48m., and to see that it leans 7° 20′ from a line perpendicular to the ecliptic; and such a line as that, through any body in the solar system, is called the poles of the ecliptic. Therefore the sun's equator is inclined 7° 20′ to the ecliptic: which means that that is the angle between them. The sun's north pole leans most towards the earth on September 13 and

his south pole on March 11. Therefore spots move apparently in straight but oblique lines across his face for about the first three weeks of June and December, but at other times in very flat semi-ellipses. Nevertheless they do not travel with the sun as if they were fixtures, but sometimes faster than that, especially when near the equator; and in latitudes above 10° the spots go slower than if they were attached to the sun: both of which are contrary to what we should expect, as the equator of course moves fastest.

Some sun spots are nearly 50,000 miles wide, and are quite visible without telescopes when the sun is dimmed by a fog. You may wonder how we can say that they are holes in the photosphere rather than dark clouds or patches upon it. If you look at a church with deep windows, the 'splays' or jambs of the windows which directly face you look equally wide on both sides; but the jambs of the windows which you see obliquely look wider on the far side than the near. And the spots on the sun have sides, less dark than the spots themselves, which appear of equal width while the spot is about the middle of the sun, but wider on the far side and narrower on the near when the spot is away from the middle. These varying sides then are almost certainly the jambs or sloping sides of the spots, which are deep holes in the photosphere. But they are not permanent holes like the volcano craters in the moon; for in some years there are none at all. They have a maximum every 11·11 years: and they decrease as they come opposite to Venus, and to Jupiter in a less degree, but apparently the earth does not affect them: the reason of all which is yet unknown. It is now considered

certain that sun spots produce magnetic disturbances on the earth, and outbreaks of aurora borealis, which is an electrical phænomenon of some kind. They are mostly confined to two zones from about 10° to 30° of latitude, answering to the region of our trade-winds.

The sun has occasionally been so much covered with spots as to diminish its light and heat sensibly.* A single spot of 50,000 miles diameter darkens a 290th of the bright surface, and the effect of this on the heat has to be calculated as at p. 54. Not that we probably lose all heat through a spot. They are generally accompanied with signs of great disturbance, and bright patches or clouds called *faculæ* are often seen on the left side of the spots, as if they had been blown out of the spot and left behind in the rotation; and changes take place quickly. But occasionally they are on the right side, or in advance of the spot.

The sun is easily proved to have an atmosphere, by the fact that it gives less light near the edges than from the middle, which a naked self-illuminated globe does not; for although you see the edges more obliquely, more surface is included within any given angle of vision, and the loss of light in one way balances the gain in the other, as you may easily see if you draw a number of lines enclosing equal angles from a very distant point to a circle. But if the globe is surrounded by an atmosphere which stops some of the light, the rays from near the edges of the globe to any distant point will evidently pass obliquely through more atmosphere than those which go out vertically from the middle of the surface. And the thinner the atmosphere is the greater is the difference of the length passed through at the

middle and the edges. A great deal of what used to be included in the term atmosphere is now otherwise designated, as we shall see presently.

Spectroscopy.—All the knowledge that we have of 'solar physics,' or of the composition of the sun, beyond the mere behaviour of spots and the existence of certain bright surroundings of the sun in total eclipses, is due to the great discovery of *spectrum analysis*, which it is therefore necessary to explain.

Wollaston in 1802, and Frauenhofer more completely after him, ascertained that the spectrum or band of rainbow colours into which a glass prism spreads or 'disperses' a ray of sunlight is crossed all over by a multitude of narrow dark lines which always occupy the same positions among the colours. Either Professor Stokes or Kirchhoff first discovered that each of them belongs to the vapour of some metal or other substance which is burnt in or before a still brighter light. But if there is no brighter light behind it to produce that contrast, the vapours themselves produce each its own line, which then appears bright with a colour of its own. So if we could see the full moon in front of the sun she would appear dark by contrast, though she looks bright by night with only the 700,000th part of the brightness of the sun. In short, the spectrum of a burning gas through a prism is not a rainbow band, but only one or more distinct lines, which appear either bright or dark according as there is a darker or a brighter light behind them.

This being established, you see at once that it affords the means of judging of the nature of any source of light, however distant, i.e. whether it is a simple incan-

descent gas, and of what kind; for then its spectrum will be its own line or lines, which can always be identified by colour and position; or there may be several gases together: or we can tell if the general source is not gaseous but has only vapours mixed with it or in front of it. The spectroscope is a series of prisms designed to spread the light more than one will, and so to make the lines more visible, for each line is not weakened as the whole spectrum is by diffusion.

The first fruit of this great discovery was the further one that the sun's light passes through the vapours of many of the common substances of the earth, proving that they exist there, and that the heat is enough to vaporise even such things as iron, which requires almost the greatest heat we can command to melt it. The next question is, where are these vapours? So many names have been given by different philosophers to the various envelopes of the sun, that it is difficult to avoid confusion, and to know whether photosphere, atmosphere, chromosphere, leucosphere, sierra, corona, all mean different things and what. No one has dealt so comprehensively with the whole subject as Mr. Proctor, in his book on the sun, and in later papers in his other books, and without constantly referring to them, the following is the best summary that I can give of the present state of the discoveries and theories on this matter.*

The 'photosphere' then, by universal concurrence, is the general light-giving or usually visible surface of the sun, which is also generally agreed to consist of what are called 'clouds,' though they are not composed of water

* See his 'Light Science,' vol. ii., 'On the Sun's true Atmosphere.'

like our clouds. This is the body in which the spots are formed as holes, and which is compared by Mr. Nasmyth, of steam-hammer fame, to willow leaves thrown together promiscuously and overlapping in every direction, and sometimes forming a bridge over the middle of a spot, Others have likened it to 'rice-grains,' and others call the granulations 'pores:' Sir W. Herschel called them 'nodules;' all evidently expressing the same idea of clouds. But what those clouds are made of nobody has yet any idea worth stating as a probable conjecture.

Outside the photosphere comes the atmosphere; which used to be described as 70,000 miles high; and there certainly is visible matter, and gaseous matter too, as high as that above the sun. But Mr. Proctor expressed the doubt whether any genuine atmosphere of quiescent gases could be allowed by the sun's great attraction to reach anything like that height. And a discovery has been since made by Professor Young of America which justifies that doubt, and has reduced the atmosphere proper to something like 300 miles. That is too small a width for any telescope to distinguish at the distance of 90 million miles, because no telescope represents two points so separated at that distance by points, but by discs which overlap each other. The larger and better the telescope the smaller the discs are, but they cannot be reduced to points by any means yet known. But for about 2 seconds before and after the beginning and end of a total eclipse of the sun, when all genuine sun light is shut off by the moon, a telescope with a spectroscope attached to it displays a series of coloured lines, making up together the usual colours, but all in separate lines like coloured threads.

The usual rainbow spectrum of the sun is gone, because the sun is hidden; but there is some collection of gases just outside it producing their own coloured lines as a spectrum for just so long as this body of gases occupies the telescope with the spectroscopic end to it, i.e. for the very short time that the moon's edge takes to traverse this atmosphere. The shortness of the time shows that this body of gases is very thin, or reaches a very little height above the photosphere—in fact not much more than 300 miles, which corresponds to nearly 3 miles round the earth. This then is the true atmosphere, and these same lines are really Frauenhofer's dark lines in the common solar spectrum; and the various metals and other things whose vapours produce them must be somewhere on the body of the sun. There seems no ground whatever for the guess which has been made, that the body of the sun may be cool though something outside it is in a state of the most tremendous heat.

Next to the atmosphere comes that which has been called 'leucosphere' though it is neither white nor spherical—'chromosphere' (or more properly chromatosphere) though it has little or no variety of colours, which both the atmosphere and photosphere have—and much more appropriately 'sierra' (Spanish for a saw) because it appears serrated, as a number of cones or flames all round the sun would do. These flames when seen in an eclipse are red, and among them there are always some 'prominences' or eruptions so much higher than the others as to reach 100,000 miles high and sometimes more, while the general average is about 5000. The spectroscope reveals that this is composed

chiefly but not solely of hydrogen, for sometimes other elements show their own bright lines also. In June 1872 during a time of unusual heat several persons observed that the sun's light had not its usual aspect, and an Italian observer found 'the chromatosphere completely invaded with the vapour of magnesium' all round the sun. Sodium lines also appear. But this differs from the atmosphere in being in a state of violent commotion, and the 'prominences' especially are proved to be projected through that dense atmosphere by some violent repulsive force with a velocity greater than a solid body would acquire in running from an infinite distance into the sun by his attraction.

Then there is the *corona*, or that radiation fully a million miles high, like what painters call a 'glory,' round the sun, which is visible only in an eclipse; for all attempts to make any of these envelopes directly visible by artificial eclipses of the sun fail by reason of the diffusion of his light by our atmosphere. But the spectroscope has been able to grapple with this difficulty also, and to show these envelopes indirectly by their representative lines without an eclipse, by diffusing and therefore weakening the general spectrum from the edge of the sun, as before mentioned, which diffusion the 'lines' do not suffer. The first suggester of this process was Mr. Huggins, the author of other remarkable spectroscopic discoveries; but the first who actually used spectroscopes powerful enough to give the expected result were Mr. Lockyer here and M. Jannsen in France, whose successes were announced on the very same day. All that appears however to be yet conclusively settled about the nature of the corona is that

it is certainly an appendage of the sun, and in no way due to the moon, or to the earth's atmosphere, as Mr. Lockyer and others for some time maintained.

The spectroscopic results do not seem to be accordant, sometimes giving a continuous rainbow spectrum, as from solid self-luminous bodies, and at other times only a few bright lines, as from a gaseous source. One of those lines is said to be identical with that of our aurora borealis, and also of the zodiacal light, to be described presently; but the nature of that also is yet unknown beyond the fact that it is due to electricity acting on some medium or other, for the magnetic needle is always disturbed by it, even in places where it is not seen. A full account of the theories or hypotheses about the corona will be found in Mr. Proctor's various works, and one on the aurora too in his 'Light Science,' vol. i. His own is that it is due to the violent expulsion or eruption of some kind of matter from the sun, not merely gaseous but solid. And if hydrogen gas can be driven through the dense atmosphere of the sun to the enormous heights measured by several observers, there can be no difficulty in supposing solid matter to be driven much farther. If the velocity of expulsion exceeds that of arrival under the sun's attraction from an infinite distance, or 380 miles a second, such matter can never return to the sun, but must run away till it comes within the attraction of some other star or planet. An unmistakable appearance of radiation certainly proves the existence of matter there, either illuminated by reflection or by incandescence of its own. Light is invisible by itself or when seen crosswise. What we call a beam of light through a hole in

a shutter or a cloud is only a beam of illuminated dust. The full moon stands in a flood of sunlight, which is dark and invisible all round her, and the sky is only light by the reflection and refraction of sunlight from the atmosphere.

If the matter, which therefore must be all round the sun as far as the corona can be seen, were simply quiescent, like an atmosphere in a state of balance between expansiveness and gravity, there would be only a generally decreasing brilliancy and no distinctly marked radiations projecting far beyond the average, as you will see in the pictures of the corona in illustrated books of astronomy. The matter must be either going or coming in various degrees of density or velocity. In either case it would be hotter and brighter both from heat and closeness together when near the sun than when farther off, and therefore we cannot decide therefrom which it is, except that if it comes from the sun it will evidently start in a state of much greater heat than if it is merely getting heated as it comes. But if matter is continually ejected from the sun, he must be fed by other matter, or his mass would decrease and the periods of the planets would increase. We shall see reason to believe that he is so fed with meteors.

There is one more solar appendage to be noticed, viz. the *zodiacal light*, which is a sort of luminous gleam stretching about as far as the earth's distance from the sun, often visible at night in the form of a lens or very large ellipse, lying obliquely to the horizon, sometimes about as strong as the Milky Way in brightness. The nature of this also is unknown. The most probable

theory seems to be that both that and the corona are composed of meteoric matter, which we shall see afterwards is now identified with comets; and that not merely the meteors, but the systems of meteors, running round and falling into, and perhaps again ejected from the sun, are innumerable. Small meteors are only seen at night, but very large ones, called *aerolites*, which are much rarer, generally fall by day; and Mr. Proctor supposes these to come from the sun, as they must evidently have come in that direction in order to fall on the sunny side of the earth; while the small ones may have been ejected towards the sun from some of the larger planets, though many of them simply revolve in elliptical orbits like the planets, but vastly more elliptical, having their perihelion very near the sun and aphelion as far off as the most distant planets. Mr. Piazzi Smyth found the spectrum of the zodiacal light in a country where it is brighter than in our atmosphere to be a 'continuous' one, indicating that its light comes from solid heated bodies and not from any gas;* and some observers say the same of the corona; but on this point there is such a diversity of testimony that it seems impossible at present to consider it as settled either way. Perhaps the bright line spectrum of the solar atmosphere, which has only lately been discovered separately, may have been confused with that of the corona. Others have identified the spectra of the corona and the zodiacal light with that of our aurora borealis, which is an electrical phænomenon certainly not due to meteors.

* See R. A. S. 'Notices,' June 1872.

THE SUN'S HEAT AND LIGHT.

The sun is the source of nearly all our heat and light to an extent which is little thought of generally. It is true that the original internal heat of the earth must be always transpiring gradually to the surface, besides its occasional outbursts through volcanoes. And some elementary substances will burn when mixed and ignited, as oxygen and hydrogen, which make the hottest flame we have, and without great care will explode and become water; and potassium thrown on water, and even on ice, ignites spontaneously. But these are all comparatively insignificant sources of heat. Wood, peat, and coal, and with it the mineral oils, either are or have been ages ago vegetables produced by the sun's heat, or partially by the then remaining superficial heat of the earth from its compression. And though the 'extraction of sunbeams out of cucumbers' is yet difficult, they certainly are extracted every time we eat one and so maintain the heat and force of our bodies, or whenever a lot of coal or wood or oil is burnt to warm or light a room or drive a steam-engine, or when grass is eaten by an ox, to be eaten again by us. Water-mills and wind-mills too derive their force from the sun, which raises water by evaporation to fall again in rain when cold, and expands the air by heating it, which contracts again when cold, and then the neighbouring air rushes in and makes a wind. Moreover the identity or mutual convertibility of heat and other forces, first propounded by Mr. Justice Grove in his 'Correlation

of Forces,' is now universally accepted; and though, as I pointed out before, gravity is not convertible, yet gravity and distance together represent the force employed from the beginning in putting things where they are, and whenever they come together by attraction they develop a corresponding force or heat. You may see in Tyndall on Heat (p. 483) that if the earth fell into the sun it would cause as much heat as the sun supplies in 94 years; and it is most likely that a great deal of the sun's heat actually is thus kept up by his attraction dragging in and as it were devouring shoals of meteors continually. So whether heat came first, and so drove all things in the universe asunder into a vast nebulous mass which has been cooling down and collecting into distinct solid bodies ever since, or distinct bodies were created and fixed at immense distances at first, with the action of gravity between them, the conclusion must be the same, that all the force now in the universe was originally created in one form or the other; and that the quantity of force or heat in the universe can no more be altered than the quantity of matter.

Perhaps the simplest way of learning that heat represents force is to take up a nail immediately after it has been hammered on an anvil or filed for a few minutes with a rough file. Tinder can be ignited by sudden compression of the air by a piston in a small tube: such machines were among the predecessors of lucifer matches. It has also been proved that the temperature of water is increased by stirring it about. Even warming your hands by rubbing them is a proof of the same proposition.

The connection between light and heat is too evident to need dwelling on. Every one knows that heat of a certain intensity produces light, though the degree of intensity varies in different bodies. Inasmuch as heat results from the stoppage of motion, or we may say from turning visible motion into heat, it is by no means improbable that heat should consist of invisible motion, a sort of tremor or vibration of the particles like that of a bell or a steel bar struck with a hammer, which produces no visible motion and yet a great noise, and vibration like that which is visible enough in a musical string. And that is the now received theory of light and heat, and is called

The Undulatory Theory; which was suggested by Huyghens and rejected by Newton for the *corpuscular* or *emission* theory, but was really established by Dr. Thomas Young of Cambridge in 1802: a discovery which Herschel said would suffice, if it stood alone, to place its author in the highest rank of scientific immortality. Lord Brougham indeed thought otherwise—at one time: for in two notorious articles in the 'Edinburgh Review,' thinking he could not err in following Newton, he denounced Dr. Young's theory and his proofs of it as paltry, absurd, contradictory, mathematically impossible, and many other things: a warning still needed against following great names implicitly. Dr. Young also completed or rather reformed the theory of the tides, which had been left imperfect by Newton and Laplace, and discovered the interpretation of Egyptian hieroglyphics and was a considerable scholar.* The undulatory theory makes light, and

* See his Life by Dean Peacock. He died in 1829, aged 56.

heat too, consist of vibrations of what is called *luminiferous æther*, vastly thinner than the air, which is supposed to pervade all space and the interstices between the atoms of all bodies (see p. 27); not that its own or any other atoms can be in contact, or it could not be elastic. The retina at the back of the eye consists of what may be roughly called a brush, of which the hairs are moved by the vibrations of light, as standing corn is by a wind. For the vibrations of this æther are supposed to set the particles of bodies themselves in vibration.

It must be admitted that there is no proof yet of the existence of such æther. Until quite lately it was thought there was, in the gradual diminution of the orbit and the time of a comet of short period called Encke's; but that proof has materially suffered by the fact lately noticed that other comets of short periods are not similarly affected.* The great argument for the undulatory theory is that it and it alone is able to account for all the known phænomena of light, while a variety of subsidiary hypotheses had to be invented to explain some of them on the corpuscular or emission theory, much like Ptolemy's and Tycho's epicycles for the solar system which could not be made to work properly in circles round the earth, and like the subsidiary hypotheses which are invented now to get over difficulties in the spontaneous evolution theory. Sound is well known to be vibrations of the common air, and cannot travel through a vacuum; a bell ringing in a vacuum does not sound, even in such an

* See R. A. S. Annual Reports of February 1873, and 1876, and Herschel's 'Familiar Lectures' on light generally.

imperfect vacuum as we can make. But we can make no vacuum against light or heat, though they can be intercepted, and the passage of heat can be impeded by some gases or vapours which are invisible. Sound will travel round a corner, though not so freely as it goes straight, but light will not, except when reflected or refracted by a change of medium. Light consists of transverse vibrations like those of a musical string, but sound consists of 'longitudinal' or backward and forward vibrations like corn in a field under the wind.

This comparison of luminous vibrations to those of a musical string must be qualified by this, that they are not in one plane, but (as we may say) all round, or in every direction crosswise, like rings expanding and contracting, or like a contraction running along an elastic tube, unless the light is what is called *polarized* by reflection or transmission through certain substances, which (as it were) squeeze the vibrations flat into one plane, or stop all except those which are in one plane; and if you try to reflect them again into a plane at any thing approaching a right angle to the original plane of incidence and reflection the light is extinguished altogether. This, like the spectroscope, but in a very inferior degree, enables us to come to some conclusions about the nature of a distant light, i.e. whether it is reflected or original; but no conclusive results respecting the sun's Corona have been obtained by it as yet.

If light vibrations went otherwise than in straight lines, and sound only in straight lines, we should have no power of judging of the distance or position of anything we cannot touch, and should also lose immensely by no sound being audible when the source of it is out of

sight. Astronomy above all things would have been impossible if light had not travelled in straight lines, except when refracted by some medium; and refraction has been called 'the bane of astronomers.'

A great deal in astronomy also depends on the velocity of light, which is in several ways ascertained to be about 185,000 miles in a second, or practically 10,000 times the velocity of the earth in its orbit; across which light travels in $16\frac{1}{4}$ min. or nearly 1000 sec. as I said at p. 86. Sound goes only 377 yards a second, about the same rate as a rifle or cannon shot; but electricity through a good conductor goes half as fast again as light, and that again is probably some kind of vibration, of which still less is known. So if the earth were shot at the sun with its present velocity and not further accelerated by attraction, and the moment of starting telegraphed by lightning, they would get the telegram there in about 5 minutes, and see a visible signal of it in 8 minutes, and have 2 months to prepare for the blow, which they would receive 15 years before they heard the original explosion. But the earth would fall into the sun by attraction, if its motion round the sun were stopped, in rather less than 65 days, and with a final velocity of 266 miles a second instead of 18.

The general connection of light and heat is obvious enough: it required a great philosopher to separate them. It is not merely that things can be moderately hot without being luminous, or that some lights have little heat; but Sir William Herschel discovered that every ray of sunlight can be split up into visible and invisible rays; the visible ones consisting of the seven

colours of the rainbow, as Newton had discovered long before, and the invisible rays consisting of heat, even hotter than the red or hottest end of the visible spectrum or band of colours which appear when light passes through a prism. Besides these there are other invisible rays beyond the violet or cool end of the spectrum, the effects of which can be made visible by chemical contrivances, as in photography; as the invisible hot rays can be concentrated by a lens or burning-glass, so as to set things on fire.

And they all come out in this order, beginning with the most refracted: chemical rays (invisible), violet, indigo, blue, green, yellow, orange, red, and then hot rays invisible. Those invisible hot rays would be hotter still, but that they are the rays which are chiefly stopped by our atmosphere and go to warm it. From an electrical light the invisible rays are much hotter than the visible ones, not having many miles of air to pass through (see Tyndall, § 308).

Here we ought to notice Mr. Crookes's remarkable discovery that light has a repulsive effect in a vacuum, but attractive in air of ordinary density. He has gone so far as to estimate the repulsive effect of sun-light at 57 tons per square mile facing the sun, independently of the question how the air may modify it. But all such estimates must only be received as such, and the discovery is too new for any theory or consequences to have been derived from it as yet. The red rays produce the greatest effect, and all the visible or proper light rays much more than the invisible ones at either end of the spectrum.

If all space is full of that which is called luminiferous

æther, a formidable result seems necessarily to follow. However thin it may be it must have some density, and the planets cannot move through it without some resistance, which must produce a sensible effect in time. It may seem odd that the effect of resistance is to shorten the period of going round the sun, as I said just now of Encke's comet. But the resistance impedes the forward motion which produces centrifugal force, or rather the tendency to go on in a straight line, out of which they are pulled by the sun's attraction. Then it is evident that the more the forward motion is resisted the more in proportion they will be pulled towards the sun, or the orbit made into a smaller ellipse, and so the period is shortened. Comets are very light bodies and may be and often are greatly disturbed by passing near planets, and Encke's comet may have been disturbed by passing through the shoals of meteors which surround the sun. But if once it is proved that the lightest comet has been accelerated by the resistance of the luminiferous æther, then it becomes certain that sooner or later all the planets must have their orbits gradually contracted, and that in fact they are moving in spirals now, getting a little nearer at every turn, and must ultimately fall into the sun, unless there is some counteracting force which we are not aware of.

The maintenance of the sun's heat is yet an unsolved problem. Several ways have been suggested in which it may possibly be maintained consistently with known facts, and even with each other, but that is the most that can be said of them. We had better first see however what amount of heat is actually supplied

by the sun in any given time. Sir J. Herschel ascertained that the light of any piece of the sun's surface is 146 times as great as that of an equal surface of the lime ball which is used under the oxyhydrogen flame; and that the heat is enough to melt 14 yards of ice all round the sun in a minute; and therefore, if an earth of ice were thrust into the sun, like a tallow candle into a pot of melted lead it would be melted in a quarter of an hour (measuring by size not weight); or the sun could go on melting four earths of ice every hour if all his heat were spent upon it.

These estimates were made by measuring the heat received by the most absorbent surface of a given size presented to the sun's direct rays for a given time within the tropics. The method is described in Tyndall on Heat (p. 73). Therefrom it was deduced that the earth receives as much heat in a year as would melt 100 feet thick of ice all over it, or boil an ocean 66* feet deep from freezing point. Another measure of the sun's heat received by the earth every minute has been given thus; except that I have reduced it to the quantity received by a square mile at the middle of the hemisphere facing the sun for the time, as the larger figures convey no appreciable idea: it would raise a block of stone 200 yards square each way, a foot a minute. But the sun's heat would take 14 years to lift the whole earth a foot. And yet the earth receives only a 2,116,000,000th part of all the sun's heat. For the size of the earth's disc, or a flat circle

* In Herschel's 'Astronomy' this has been printed as 66 *miles* through various editions, and the mistake has been naturally followed in other books, including some of the earlier editions of this.

of the earth's diameter, is about $\pi \times 4000^2$ miles, but the surface of a sphere at the earth's distance from the sun is $4\pi \times 92{,}000{,}000^2$, which is about 2116 million times the earth's disc. And all the planets together only get a 227,000,000th of the sun's heat, as you may find by adding together 4 times their distances^2, each $\div$ its own radius^2. Therefore, so far from his having occasion, as we may say, to economise his heat, he makes 227 millions as much as his planets get and radiates the rest away into space.

Latent heat.—These figures require a little explanation. We saw at p. 55 that ice may be said to contain 143° of cold beyond water at just freezing-point; or that is the latent heat of water. Again, boiling water will swallow up 980° more heat before it is all turned into steam, and so that is the latent heat of steam. Consequently it takes nearly half as much heat to melt a piece of ice as to boil it, and eight times as much to boil it all away into steam (1123°) as to melt it, or six times as much as to make ice-cold water boil. But for this remarkable provision of the Creator, and another to be noticed presently, all our rivers and ponds and cisterns would often be frozen to the bottom in a single night, and remain so as long as the air is a single degree below freezing; and as soon as it rose the least above 32° all the ice and snow that had fallen would melt suddenly, and produce destructive floods; and steam would be unmanageable and almost useless. If you melt a pot of lead it falls into fluid nearly all at once, not like a lump of ice put into warm water; and in like manner lead, or cast iron, or mercury (at 40° below zero), get solid nearly all at once, because

they have so much less latent heat than water; that of lead is only 9°.

Specific heat.—Two other causes prevent water, and especially water only open at the top, from freezing too quickly. One is that it takes much more heat to raise a pound of water 1° of a thermometer than a pound of anything else: thirty times as much as mercury or lead, solid or fluid; twice as much as oil, and half as much again as spirits of wine. Therefore they call *the specific heat** of water thirty times that of mercury, and so on. And again, the steam of water expands far more than any other steam. The other reason is that water (like antimony,† iron, and a few other things, which crystallise in freezing) expands instead of contracting a little before it freezes, as well as when it is heated, and so the ice and very cold water stay at the top. Besides that, it *conducts* heat very slowly. The hot water in boiling rises bodily, and in that way heat is soon *conveyed* through the mass. Therefore the sea gets heated from above slowly, and gives out a great deal of heat in cooling after it has travelled into colder regions, and so equalises temperature.

We may reject at once any idea of the sun's heat being supplied by the burning of any of his own substance. A sun of coal burning fast enough to produce that quantity of heat would be burnt to ashes in 5000 years if supplied with oxygen enough to burn it. It is true that the sun may be composed of something containing more heat than coal, which is only taken as an

* In some books latent heat is confounded with specific, but they are quite different.

† Types could not be cast fit to use till that was discovered.

illustration. But we have no reason to suppose that it is, or that any kind of matter exists capable of maintaining the sun's heat by combustion for any period worth considering. Guessing that there may be, and that the law of conservation of force may not exist there, and that the same fuel may be burnt over and over again (which has been seriously suggested), is not philosophy, but the invention of a new kind of universe and laws of nature made to order. A less impossible theory was propounded by Mr. Mathieu Williams in his ingenious book on the 'Fuel of the Sun,' which assumed that fuel to be either atmospheric or hydrogen gas, which he thought may (or rather attempted to prove, must) exist in a very attenuated state throughout space, and be gathered in by attraction, and as it were devoured by the sun as he moves over nearly the width of the earth's orbit in a year. But these propositions met with no reception among mathematicians; and I only refer to the theory because it was so ingeniously supported by a very able man. Of course it was suggested by the fact that hydrogen is proved to exist in a state of inflammation round the sun, as I have stated; and there are occasional outbursts of inflamed hydrogen in the stars, but those are only temporary.

The only two theories of solar heat production which receive any favour from astronomers at present are the meteoric and the contractive. The meteoric one has been already referred to under the theory that the corona and the zodiacal light may represent the confluence of the infinite number of meteoric streams which exist; or possibly one of them may be the confluence

of such streams reaching the sun, and the other the counter streams ejected, as there is undoubted evidence of there being some ejective force in the sun which prevails greatly over his attractive force in producing the sierra and its prominences. It has been calculated that a rain or hail of meteors as heavy as granite (and they are generally heavier) falling 12 feet thick all round the sun with the velocity they would acquire in coming straight from an infinite distance (380 miles a second) would maintain the actual heat. Or if they fell with the velocity of small planets dragged out of their orbits in consequence of some gradual obstruction to their progress, which would be ultimately 240 miles per second, it would require 24 feet thick of such meteoric hail to keep up the sun's heat for a year.

But it appears to me that there is a difficulty in the way of that theory which requires at least some supplementary hypothesis to dispose of it. Bearing in mind that the specific gravity of granite is about $2\frac{1}{2}$ that of the sun, a shell of granite 8 yards thick would be an addition of about a 12,380,000th of the sun's present mass annually; or the sun must have been in that proportion lighter every year reckoning backwards; and since the length of the year must vary inversely as the $\sqrt{\text{sun's}}$ mass (p. 84) it must have been always shortening a 24,760,000th annually. That is not quite 1·3 seconds, it is true; but an accumulation of 1·3 seconds a year comes to a good deal in the 2500 years since tolerably exact observations began to be recorded, or at least since total eclipses were identified with historical events occurring at known epochs. The sum of

any great number of terms in arithmetical progression is practically their common difference × half the square of the number of terms,* which would amount to 47 days. Or again the mere sum of 1·3 seconds a year in 2000 years since the time of Hipparchus would have made a difference of 43 minutes in the length of the year by this time; and it is certain that there is no difference of half as many seconds (see p. 76).

I do not know whether it is this difficulty that caused Sir W. Thomson, one of the authors of that theory, to give it up, though others still adhere to it. If the sun is ejecting meteors by some other force at the same rate that he is receiving them, that force has itself to be maintained somehow and is only an equivalent of heat in some other form; and therefore that leaves the difficulty where it was. If the meteors were all before their absorption within the earth's orbit, forming a sort of spherical extension of the sun, it is true that their joint attraction on the earth would be the same as after they have fallen into the sun. But I have seen no suggestion that that is so, and many meteor systems, especially the two largest that we know of, have orbits reaching far beyond the earth's.

The other theory is that the sun is still contracting under his own attraction; and it is calculated that the shrinking of his diameter by a 10,000th or 84½ miles would generate heat enough for 2000 years. There seems no particular reason to suppose that the sun or

* Properly it is $\frac{1{\cdot}3 \times 2500 \times 2499}{2}$, which only differs by a 2500th part from the simpler formula, which has the advantage of reminding us that the accumulation practically increases as the square of the number of terms of a long arithmetical progression.

planets have reached their maximum density by contraction; but on the other hand there is no evidence that they are contracting. No period in astronomy is more constant than the length of our day, and yet if the earth had shrunk the smallest conceivable quantity the day would have been very sensibly shortened by this time, by the diminution of 'the moment of inertia' of rotation, which makes a large thin grindstone harder to turn than a short thick one of the same weight. It is possible that this may be just balanced by the retardation which we shall see must be due to the friction of the tides upon the earth. But the heat caused by contraction would be almost entirely internal, and we know by experience on the earth how slowly that heat transpires, and what we want to account for in the sun is not internal but superficial heat. Therefore this theory also is a long way from being either verified by observation or free from difficulty. And whatever may be the probabilities in favour of this or the meteoric theory, it cannot be said that any thoroughly satisfactory mode of maintaining the sun's heat and light has yet been invented upon earth.

I suspect that if many other things which have become familiar to us by observation had been so distant that we could only see their effects, as we do the sun's heat and his appearance, philosophers would have been quite as much puzzled to invent the means of doing them, though some of them now see no difficulty in what they call 'nature' making birds fly and trees grow and all living things reproduce their like, and men able to command the world and discover the secrets of nature, by 'spontaneous development' from oysters and sponges;

forgetting that 'nature' is a mere word, meaning only the ordinary course of things, and that the real problem to be solved is, why the ordinary course of things is what it is, or what power makes it so. But this question is assuming so much importance, and the language used by some modern philosophers about it is so evasive of the real difficulties of the theory which they constantly insinuate rather than assert, that I have added a special note upon it at the end of the book.

CHAPTER III.

THE MOON.

If you ask half-a-dozen people how big the moon looks, you will probably get as many different answers, varying perhaps from a shilling to a large round table. The truth is that the question has no meaning, because a shilling near the eye looks as large, or makes the same angle with the eye, as a plate farther off, or as the moon farther still. But then some people say the moon looks bigger than the sun, others smaller, and others about the same size; and that has a meaning, and can be wrong or right; for we can compare the angle which each of them subtends at the eye, and say which has the *real appearance* (quite a different thing from the reality) of being the largest. Without any measuring at all there happens to be a ready way of knowing that they really appear about the same size, sometimes one a little larger, and sometimes the other. For the moon quite covers the sun in a *total* eclipse, but leaves a bright ring all round it in an *annular* eclipse. In the former she appears at least as large as the sun, being at her nearest to the earth: in the latter a little less, being farther off.

But as soon as we take the distance of the sun and moon into account, we begin to see what a little thing the moon is. The sun is 385 times farther off at their

mean distances, and his real diameter is 396 times the moon's and therefore his real bulk 396^3 or above sixty million times as great. Her diameter is only 2160 miles, and therefore the earth is 49 times as large, as you may easily calculate from cubing the diameters; and her mean distance 238,820, or 30·14 times the earth's greatest diameter. Her mean apparent diameter is 31′ 6″ while the sun's is 32′ 4″, these of course varying a little both ways with the variations of distance. It may help you to realize these dimensions to say that if you take a ball an inch wide for the earth, the moon will be about the size of a pea 30 inches off, and the sun a globe of 9 feet diameter 320 yards off. The earth is the densest of them all; for we have seen that its specific gravity is nearly 4 times that of the sun, and it is 1·7 times that of the moon, or the moon's density is ·6 of the earth's; for the earth is found to weigh 81·5 times, and therefore the sun 26 million times as much as the moon. It is remarkable that the moon's distance is 110 times her diameter, and the sun's distance nearly the same multiple of his (108), and that again is 108 times the earth's diameter, as noticed at p. 80. But these are mere accidental coincidences with no physical reason for them.

These are all mean or average distances and apparent diameters. The moon's orbit is much more eccentric than the earth's, and the eccentricity itself varies much more quickly, from ·044 to ·066, the mean being an 18th, or more than 3 times that of the earth's orbit now. When the eccentricity is greatest the moon's distance at *perigee* or when nearest the earth is little more than 221,593 miles, and her greatest distance 252,948; and

the corresponding apparent diameters are 33′ 30″ and 29′ 21″ * when corrected for a certain optical excess called *irradiation*, which makes a bright body on a dark ground look a little larger than it ought; as you may see in the outside of a thin crescent moon evidently extending beyond the ends of the internal semi-ellipse. The true apparent diameter is got by occultations of stars behind the moon's dark side; for the places of the star and of the moon's centre are known, and their distance at occultation is the moon's apparent radius.

You may observe that the mean distance and apparent diameter before given are not half way between the greatest and least: the reason is that they belong to temporary orbits of three different eccentricities, and the mean distance, SB in the figure at p. 48, of the mean ellipse is not the average of the greatest distance SA of the longest and narrowest ellipse and the least distance SD of the shortest and widest ellipse. I only add to these figures that the moon's mean parallax, or earth's apparent radius (not diameter) seen from the moon, is 57′ 3″. We shall see afterwards how this, which is the real measure of the distance, is ascertained. Since the size of the disc varies as the square of the diameter, it is easy to calculate that the largest apparent disc of the moon is very nearly a third larger, and therefore gives us a third more light than the smallest.

The moon is so near that probably some persons walk as far in their lives, as 13 miles a day for 50 years would do it. She is at any rate near enough for us to see with telescopes that her surface is very

* I take these figures and many others from Mr. Proctor's 'Moon' as the latest good authority on the subject.

different from the earth's. It is covered in many places with craters of old burnt-out volcanoes, far larger than any on the earth, and with the heaps of ashes which were thrown out forming mountainous rings around them. Indeed it has been lately doubted whether all volcanic action is extinct, as one crater called Linnè, 7 miles wide, has been thought to have been raised from a hollow into a hill; and some other changes have been suspected; but it is by no means certain that these are not all optical effects. There are also wide plains called seas: but not of water, for there are no signs of any; nor of any atmosphere affecting the light near the edges of the moon, as there are in the sun and planets. The regularity and distance of the rings from their craters, and the height of the mountains, if they are so formed, would depend on the volcanic force and the inclination at which the matter was thrown out, as there was no wind to stop or spread the ashes; and that force would be practically greater on account of the weakness of gravity and the quicker cooling and contraction, from the moon's smallness and the want of any atmosphere to check radiation away of the heat into space. But rings could not he formed unless all the matter was ejected at the same inclination all round the crater, which is most improbable. Mr. Mathieu Williams, and indeed others before him in less detail, have remarked the similarity of the moon's surface, including the volcanic action of former times, to what now takes place in masses of melted iron in cooling.*

A telescope which magnifies 1000 times shows the

* See R. A. S. 'Notices,' March 1873, or Proctor's 'Moon,' p. 354.

moon as we should see it 233 miles off without a telescope; for we have to measure between the two surfaces, not the centres. A town a mile wide would look nearly half as wide as the moon does, and a building of the size of York Minster would look larger than Venus. But no shapes of any such objects could be distinguished if they existed, even by the largest telescopes. We see the shadows of the mountains when they are turned sideways to the sun.

Rotation of the Moon.—It is one of the peculiarities of the moon, and (as far as we can see) of the moons belonging to other planets also, to keep the same face always towards its 'primary,' as if she were set fast upon a stick reaching to the earth. This is accurately described by saying that she turns on her axis exactly in the same time as she rovolves round the earth. You may think at first that the moon's keeping the same face to us proves that she does not turn on her axis at all; and every now and then somebody undertakes to prove that all the astronomers are wrong in saying that the moon rotates or turns upon her axis; as others undertake to prove that all the mathematicians in the world are wrong in saying that the diameter of a circle bears no exact proportion to its circumference or area.* I know it is as hopeless to

* I mean no proportion measurable by any finite number of figures, or by any geometrical construction with rule and compasses. Mathematicians know very well that it can be done mechanically by a cycloid, or the curve traced by any point in the circumference of a wheel rolling on a straight rail; for the base of the cycloid evidently = the circumference of the wheel, and the parallelogram which contains the cycloid has an area = four times that of the circle. See De Morgan's 'Budget of Paradoxes,' on this subject, *passim*.

convince any of that race of 'paradoxers' as it was the man who insisted that his head had been twisted and looked backwards.

The confusion really arises from the very common cause, of persons who are not versed in a particular science (whatever it may be) taking upon themselves to attach a meaning of their own to a word which never has that meaning among those who are versed in it. A man may choose to say that in his opinion 'mass' ought to mean size and not weight; but that does not make mathematicians wrong for using it as meaning weight, under the conditions explained at p. 22. So he may choose to say that in his opinion 'rotation' does not mean absolute rotation but relative. But astronomers invariably mean by it absolute rotation, or the successive presentation of every side of a body, except its top and bottom (speaking popularly), i.e. of every side round an axis, in some given direction in space, disregarding any other motion it may have. Then comes a 'paradoxer' and says, 'I won't call it rotation if the body has some other motion which enables me, standing in a particular place and turning myself round, only to see one side of it.' If such persons are worth arguing with they should first be called upon to give their own definition of rotation; 99 out of 100 of them can give no rational definition at all; and the one who can, if not standing out for mere obstinacy, will see that his definition is essentially different from the astronomical one, and that those who start with different definitions of the same word cannot possibly agree about its use; and he may be sure that astronomers and mathematicians

will not change the meaning of their language to please him.

Nevertheless for the information of those who wish to learn I will give a few of the innumerable proofs that can be given of the moon's rotation.

First then the deniers of it forget, or do not know, that if they were on the sun, or anywhere else in or near the infinite plane of the ecliptic, except just here inside the moon's orbit, they would see opposite sides of her every fortnight; and the proposition that she does not turn would then appear as absurd to them as the contrary does now. People on the moon itself, if they knew the real motions of the sun, earth, and moon, would be infinitely surprised at being told that they have a day and night without rotation. The earth is to them what the moon is to us, both going round the centre of gravity of both, like a large and a small ball at opposite ends of a stick whirled in the air. If we had happened to have 12⅓ days in the year instead of 365 we should have thought it very strange to be told that the earth does not rotate because the moon would then see only one side of the earth. And the cases are absolutely identical. The moon presents all her sides to any star 13·37 times a year, but to the sun 12·37 times: i.e. she rotates absolutely 13·37, and relatively 12·37 times.

Perhaps the mistake arises from thinking of the moon's axis as a wire stuck through it, and forgetting that if it were, the question would remain whether the wire is turning on its own mathematical axis or not. If you put a wire upright through a ball which has one side painted white, and bend it into a horizontal

tail, which you can hold always pointing to one side of the room, while you carry the ball round you with the white side always towards you, it will plainly enough turn on the axis because the axis has been kept from turning. But if you keep the tail of the wire always towards you, the ball does not turn on that axis, because that has turned just as much on its own mathematical axis.

Or you may lay a book on a revolving table, at some distance from the centre. The deniers of the moon's rotations say the book does not turn on its own axis because it keeps the same face to the centre of the table when you turn it round. But that can have nothing to do with its distance from the centre. Keep gently moving it nearer to the centre while the table revolves, and at last it will be visibly turning on its own axis and nothing else. Then at what moment did it suddenly start from no rotation into a rotation at the same rate as the table? Many more illustrations may be and have been given, but they would all be equally superfluous both for those who cannot or will not understand these, and for those who can and will.

So much for the fact of the moon's rotation. But you may prove also that a force of rotation must have been originally impressed upon it to make it rotate even at that slow rate of once a month. Hang a globe or anything else, so marked as to distinguish the sides, by a thin string from the end of a stick held in your hand, till it has got quite steady: then turn round, carrying the stick with you so as not to shake the globe. You will find that by the time you have gone half round, the side which was facing you has got farthest off,

but still faces the same side of the room. Why? Because there has been no force impressed on it to make it turn, and nothing ever moves except in obedience to some force. Again, if a globe floats in a basin of water it will not turn when the basin is turned, for the same reason. This was an experiment of Galileo's. The globe should be large, for the friction of the water will affect a small one.

Libration.—It is not strictly true that the moon always keeps exactly the same face to us; for she apparently rolls a little in her orbit, so as sometimes to show a little more on the right side, and sometimes on the left. That is called *libration;* and it has been ingeniously used to take stereoscopic photographs of the moon, which show the heights of the mountains and the depths of the volcanoes. For a stereoscopic view of anything is got by taking two pictures of it, one seeing a little more of the right side and the other of the left, just as your two eyes do, as you may see by winking them alternately while you look at something not far off; and that is the way the eyes judge of distance, when it is not too great for any sensible difference in the view which each eye takes.

The reason of the moon appearing so to roll is, that she does not move in her elliptical orbit with uniform velocity, but does turn on her axis uniformly, and so the two motions do not quite keep pace with each other. She is sometimes $7° 20'$ before, and sometimes $7° 20'$ behind the longitude she would have if she moved uniformly: that being her greatest 'equation of the centre,' or difference of true and mean 'anomaly' or distance from perigee, when the eccentricity of the

orbit is also greatest. And the libration is the same as if the moon stood still, rotating uniformly, and the earth went round her, and at the same rate on the average, but sometimes 7° 20′ before and behind its mean place. Consequently that is the amount of the *libration in longitude*, each way; and we see on the whole 14° 40′ beyond the half of the moon's surface; and indeed 15° 30′ by reason of the other inequalities in her motion which will be described hereafter.

Besides this, the earth's rotation carries us to a different point of sight from moon-rise to moon-setting, or from the western to the eastern boundary of the earth, which makes an angle of libration = the earth's diameter ÷ the moon's distance, or nearly 2°, to observers at the equator; but observations of objects near rising and setting are unsatisfactory on account of refraction of the atmosphere, and far from the equator the change of position by rotation is of course less than the earth's diameter.

But the polar axis also leans 6° 39′ from the perpendicular to her orbit, and sometimes one of her poles leans towards us and sometimes the other; and so we see also 6° 39′ beyond each pole alternately. This is called the *libration in latitude.* Now the area of a *lune*, or slice of surface of a globe between two meridians, is evidently measured by their difference of longitude; and therefore the surface disclosed by the greatest monthly and daily librations in longitude bears the same proportion to the whole moon's surface as 17° 30′ does to 360°, and that by libration in latitude as 13° 18′. But they overlap a little, and the result is that all the librations together enable us to

see nearly ·6, or one fifth more than half of her whole surface at one time or another.

Though the moon goes round the earth, it is no less true that the earth goes round the moon, only in an orbit as much smaller as the earth is heavier; that is, 81·5 times. The point between them which remains fixed, if we forget their joint motion round the sun, is their centre of gravity, which is 81·5 times nearer the centre of the earth than of the moon, because the earth is so much heavier than the moon. Therefore the c. g. is one 82·5th of the moon's distance, or 2895 miles, from the earth's centre. By this we shall hereafter measure the distance of the sun, besides other methods. So if you fix two balls on the ends of a stick five feet long, one four times as heavy as the other, and whirl it into the air, they will turn round the centre of gravity, four feet from the light ball and one foot from the heavy one.

The cause of the moon keeping her distance from the earth is simply the centrifugal force of both of them. If the light ball just now described can slide along the stick, but is connected with the other by a string, and you whirl them into the air, they will keep the string stretched by their centrifugal force. The same is the reason why the earth and all the planets keep their distance from the sun, subject to the variations due to elliptic motion, as I shall explain hereafter.

Calculations of the moon's motion are simplified by 'reducing the earth to rest,' as it is called: which might be done by assuming another equal moon at the same distance on the opposite side of the earth, so that

the common centre of gravity and of motion would always be the earth's centre. But this hypothesis would make the mass of the whole system too great, and would increase the time of its revolution; and the more correct way of reducing the earth to rest is to suppose the mass of the moon transferred to the earth, leaving her only an empty shell, as explained at p. 25. But whenever we have to deal with the moon's attraction, as on the equatorial protuberance of the earth in precession and nutation, or on the water of the earth in the tides, we must of course deal with her real mass, though we may still consider the earth at rest, except as to rotation, because that alone affects those disturbances.

We are so much in the habit of regarding her as a mere appendage of the earth that we forget that the earth is the moon's moon. To men in the moon (if there were any) knowing as much astronomy as we did until quite modern times, the earth would be merely a moon of nearly 4 times the diameter of our moon to us, or of the sun to both of us: giving them about 13 times as much light as the moon gives us, if the surfaces are equally reflective. If they are as ignorant of lunar rotation as some people are here, they will say that their much larger but variable and much weaker luminary (the earth) stands still like themselves and that the smaller sun and stars go round them both; but they must see also that the earth rotates above 29 times in one of their days. If tney have discovered that it is not the sun and stars that go round them, but their own globe that rotates, to make their day and night, they will be struck with the odd coincidence

that their large-looking moon goes round them exactly in the mean time of their own rotation. When their knowledge advanced still farther to a discovery of real sizes and distances, they might still for convenience treat the earth's orbit round them as we do the moon's, referring all the mutual motion to the earth as an empty shell of no weight and considering their globe only with reference to its own motion round the sun, as we do in dealing with ours.

The Moon's real Orbit.—Some books of elementary astronomy represent the moon's path in space as being like the curve traced on a wall by a pencil stuck in a wheel rolling along the ground: which is a wavy line called a 'trochoid,' alternately convex and concave if the pencil is not at the circumference, and a cycloid if it is, which has a cusp or pointed turn at every lowest point. Others represent it as a looped spiral, and actually give a picture of it so. The real orbit is nothing of the kind, but is simply the earth's orbit (or more exactly that of the common c. g.), with 25 alternate very slight depressions and elevations of its concavity towards the sun, at each new and full moon, there being nearly 12½ (12·37) lunations in a year. From one half moon or quadrature, or crossing of the earth's orbit by the moon, to the next is 14° 33′, which is 23,365,000 miles with the radius of 92 millions. You must take it as easily found by trigonometry that such an arc bends outwards 740,000 miles beyond its chord or the straight line joining its two ends. But the moon is only 236,000 miles from the c. g. of earth and moon; and therefore when she is nearest the sun she is is still 504,000 beyond the chord; for her path

through the new moon fortnight is only rather less concave to the sun than the earth's orbit. But if she went round the earth in a fortnight the path of a new moon would be just convex to the sun; for she would then be carried 184,100 miles outwards with the earth and 236,000 inwards by going round the earth. Her path in the full moon fortnight, being all beyond the earth's orbit, is evidently rather more concave than that. It can easily be shown by a little geometry to be also concave to the sun at quadratures and everywhere; but the difficulty to most people is in understanding that it can be concave to the sun at new moon. If you draw a piece of a circle with a radius of 20 inches, the arc of $14^\circ\ 33'$ will be 5 inches; and the chord will be ·16 of an inch from the middle of the arc. The 236,000 miles of moon's distance, reduced to the same scale, is only ·06 of an inch; but the difference from the earth's orbit is hardly visible in a circle of that size.

Revolving elliptic orbit.—In all calculations of the moon's visible place her orbit round the earth may be considered an ellipse, with the earth standing in the focus, and the sun revolving round them both. And this elliptic orbit, or its major axis, or line of *apsides*, or *perigee* and *apogee*, or least and greatest distances from the earth, revolves in 8 years and 310·575 days, going forwards; whereas the earth's perihelion takes 110,880 years to revolve, independently of the precession of the equinoxes, which go the other way. An anomalistic month, or the time of moon's return to perigee, is 27.5546 days.

You may easily see a revolving ellipse if you hang a weight by a long string to a hook in the ceiling and

send it swinging in any direction, except straight across like a pendulum or quite round in a circle, which indeed you would find it difficult to do. If you stand near the narrow part of the ellipse where the weight passes by you, and stay there, the ellipse will wheel round and the weight will hit you before long. The only difference is that the centre of force in this ellipse is not in the focus, but the centre, the force being the tendency of the weight towards the vertical or lowest point; and it is nearly in direct proportion to its distance from that point, instead of the inverse proportion of the square of the distance. This ellipse wheels forward, or in the same direction as the weight goes, because the force does not quite vary as the distance; and the lunar ellipse similarly wheels forward because the moon's attraction to the earth is a little diminished by the sun, as will be explained hereafter.

The moon's axis, like the earth's, does not stand upright to her orbit, but as I said just now, leans 6° 39′ from upright: that is, the moon's equator is so much inclined to the plane of her orbit. And again the moon's orbit round the earth is inclined 5° 9′ to the apparent orbit of the sun round the earth, or the ecliptic. So we have now got one more plane and one more inclination to consider than we had in dealing with the earth's equator. It is however simplified by the fact that the crossing of the moon's orbit and equator always coincides with the crossing of her orbit and the ecliptic; though that line, which is called the line of *nodes*, revolves backwards 19° 21′ a year, making a sidereal revolution in 18·6 years, or 6793·391 days, or 223 average calendar months, just as the equinoctial

points ♈ and ♎, the nodes of the earth's equator and ecliptic, revolve round once in 25,868 years. The node at which the moon goes up to the north of the ecliptic is called the ascending node ☊, and the one where she goes down again to the south is the descending node ☋. It is an odd coincidence that the node returns to conjunction with the sun and moon in 223 lunar months; a nodical month, or time of moon's return to the same node, being 27·21222 days.

Moreover it is remarkable that the ascending node of her orbit is the descending node of her equator: or at the moment when she rises in her orbit from the ecliptic at the inclination of 5° 9′, her equator is inclined 1° 30′ to it the other way. Therefore the ecliptic always lies between the plane of the moon's orbit and the plane of her equator, making the angle 5° 9′ with the former and 1° 30′ with the latter.

Shape of the Moon.—Some of the stereoscopes of the moon taken at opposite librations present the appearance of a prolate spheroid pointing towards the earth. This may be an optical illusion or defect; but other observations appear to show that she actually is like an egg with the small end towards the earth and the large end behind. If you look straight endwise at an egg, with one eye, you cannot tell that it is not spherical, but you can if you see it a little obliquely. By a careful measuring of angles you might find its exact dimensions, and the more obliquely you can see it the better. It appears that the oblique profile of the moon does show her to be egg-shaped; and so much, that unless her density is irregular, her centre of gravity is 66 miles nearer the back than the front; or

33 miles beyond the middle of her longest diameter. Hansen said that this is also requisite to explain some of the inequalities of the moon's motion: but Professor Adams is not satisfied of that.

And then this consequence would follow: the average mass of the long half of the moon is farther from her c. g., round which she must rotate, than the mass of the short half; and the earth's attraction on each half, being inversely as the square of its distance from them, will be greatest when the long half points to the earth: which will tend to keep it there; though not strongly enough to prevent the libration of a few degrees, which is due to the moon's force of rotation.

Even if she is not egg-shaped, with a large and small end and the c. g. beyond the middle of her length, but merely a prolate spheroid with the c. g. in the middle, still the earth's attraction would tend to keep the long axis always pointing to us, if it were otherwise inclined to deviate with no great force of rotation. For suppose such a prolate spheroid turned a little aside; the earth's attraction is rather greater on the nearer end of the spheroid than on the farther end, and therefore tends to bring the axis back again to the line pointing to the earth; though not so strongly as with the egg-shape, where the near and small end is still farther from the c. g., and therefore the difference between the two attractions is still greater. Newton calculated that the moon ought to be 178 feet longer than she is broad, assuming that she was originally fluid; and we shall see when we come to the tides that that estimate ought to be rather more than doubled, because the moon is much lighter than he supposed, and therefore her

attraction on her own mass is so much less compared with the earth's. This however is far from accounting for a distortion which places the c. g. 66 miles nearer the back than the front of the moon, nor has any way of accounting for it been suggested, and the fact is not yet considered certain by astronomers.

The question whether there is either air or water in the moon is connected with her shape. If she is egg-shaped (I do not say 'oval' because that has acquired the incorrect meaning of having two equal small ends) with the c. g. nearer the back, it was said by Herschel and naturally followed by others, that any moderate quantity of air and water would run to the back to get nearest to the c. g. But Mr. Proctor pointed out that that could not be so; for such a figure (a sphere with a cap added to it in front), and *à fortiori* a prolate spheroid, evidently has its c. g. farther from the back than the sides, not nearer.* Consequently if the moon has any atmosphere, it is in the best and not the worst position for showing its effects, by refracting any rays of the sun or stars which come through it, grazing the real surface of the moon. But it is certain that no such refraction takes place, for stars are 'occulted' or eclipsed by the dark side of the moon in a moment, whereas they would be gradually dimmed if they passed behind an atmosphere. And if there were any water, there must be an atmosphere of vapour at least, raised by the sun's heat, and that obstructs rays a great deal more than dry atmosphere, which hardly does so at all, though it refracts them. Therefore the moon has neither air nor water.

* R. A. S. 'Notices,' xxxiii. 350, and Proctor's 'Moon,' p. 300.

Heat of the Moon.—A consequence of the moon having neither air nor water is that the sun's heat falls upon every part of her in succession for a fortnight, unmitigated by atmosphere; and for the other fortnight, having no atmosphere to prevent the heat from radiating away, she is exposed to the 'cold of space' diminished only by what the heated ground retains for a time. Various estimates have been made of the extreme of these differences, reaching as high as 500° of heat and nearly as much below zero of cold, or far beyond the difference between ice and melted lead; and though there is no water for the frost to act on, the alternate expansion and contraction of the rocks by these variations of heat every month must tend to disintegrate them. But as a mere sand hill will stand at a slope of about 1 to 1 (as engineers call it) it does not follow that the lunar hills would be levelled in time, as they would be here by rain, frost, wind, and gravity, but for volcanic action, either visible or latent, which as Sir J. Herschel showed, is always at work in altering the levels of the earth, sometimes up and down again within the times of history.

The moon has no light of her own, and only reflects the sun's. Dr. Wollaston made out that full moonlight is 800,000 times weaker than sunlight, which he found equal to 5563 wax candles at the distance of a foot, while the moon was only the 144th of a candle.* Therefore, as the moon fills about the 240,000th of the visible hemisphere, we should get much less light from a whole sky full of moons than we do from the unclouded sun. Later experiments have made the full

* 'Philosophical Transactions' of 1829, p. 20.

moon's light a 618,000th of the sun's; but all such results must be very vague, depending on ocular judgment of equal illumination, with all the errors multiplied enormously.

Moonshine produces no visible rise of a thermometer, even when concentrated by a very large concave mirror into the greatest possible intensity upon it. But Lord Rosse has at last succeeded by the more delicate apparatus called a thermopile in finding that the moon does reflect a little heat, which sensibly increases with the width of her illumination. The full moon however causes cold indirectly: for it is said to drive away clouds, and a light night is always colder than a cloudy one (when other things are equal), because the heat of the earth radiates away into space instead of being kept in by the clouds. But this has lately been disputed.

Phases of the Moon are nothing more than the Greek word for her appearances or faces. One half of her is always illuminated by the sun, but the illuminated hemisphere is constantly changing by her monthly rotation. The effect is just the same whether we consider the moon moving round the earth, or the earth round the moon (as they do mutually), or the earth and moon fixed and the sun going round them both. In either case we should see a bright hemispherical cap wheeling round the moon on its diameter, always facing the sun. When the sun is straight beyond the moon of course we see nothing of the bright cap, and that is called new moon, because in a day or two we begin to see the right edge of the cap as a very narrow crescent, with the horns to the left, which increases every night, and is called the moon *waxing*. In a week we see just half the

bright hemisphere, or a half moon, and that is called the first quarter—i.e. of a lunation; and then more than half, till we see the whole or a full moon, the sun being then opposite to the moon or behind us. Then the right side begins to *wane* or darken towards half moon again; and then during the last quarter there is again a crescent, but with the horns to the right.

The inner edge or the boundary between the bright and dark part of the moon is called the 'terminator;' and as it is an oblique view of a semicircle it is always a semi-ellipse, except exactly at half moons, when it is a straight line. When the bright part is convex on both sides the moon is called *gibbous*. In the figure at p. 48 the shaded part of the ellipse represents a crescent, and the unshaded part of a gibbous moon. The bright part always looks a little too big, from the cause before mentioned called irradiation, which enlarges the full moon about 2″ all round.

Some persons are foolish enough to predict bad weather when 'the moon is lying on her back,' i.e. when the hollow part of the crescent is turned upwards. If that had anything to do with it, astronomers might predict the weather very easily, since the posture of the crescent depends on the moon's position in her orbit. The 'horns' are the points of intersection of the edge of the illuminated hemisphere and of the hemisphere facing the earth, and the line joining them may be called the axis of illumination, which is therefore perpendicular to the plane of the sun moon and earth for the time. That plane is not quite the ecliptic, but a shifting one, because the moon's orbit deviates 5° 9′ from the ecliptic. If the moon moved in the plane of

the equator, she would always rise with the upper horn to the left of the lower, and the contrary in setting, and be upright on the meridian. When she is rising, and is at the same time near her own ascending node and near the earth's ascending node ♈ also, the obliquity of her orbit to the equator in the upward direction is greatest, and so her upper horn is thrown back, or north-eastward, as much as possible, and the axis of illumination may be almost horizontal. At that time the upper horn leans back even when she is on the meridian. And this is true to a greater or less extent whenever the moon is near ♈, but greatest when her ascending node is also there; and the converse when she is setting; and the converse of all these when she is in ♎. It is not so much noticed when she is gibbous as when she is a crescent, though the effect is really just the same.

A day or two before or after new moon you may see what is called 'the new moon with the old one in her arms,' or a bright narrow crescent, with the rest of the circle just light enough to be seen. That is the reflection of the earthshine back from the moon. When she is farther from new, the earthshine appears less and the proper moonshine stronger, and so the earthshine is too weak to be seen.

As the moon has no light of her own, but only reflects the sun's light to us, so we do the same to her. If there were any men in the moon they would see corresponding phases of the earth, which would be full to them when the moon is new to us; and the earth would appear to them nearly thirteen times as large as the moon does to us: it would be sixteen if the diameter of the earth were quite four times that of the moon; for

the apparent size of a globe, which is called its *disc*, being a great circle of the globe, varies as the square of the diameter, while the real size or solid content varies as the cube, as I explained before.

Periods of the Moon.—'Period' means the time of performing a journey round. The average time from one new or full moon to another is 29d. 12h. 44m. 2·87s., or 29·5306 days, or 708·734 hours. But to reach a second new moon, or the line between the earth and sun, the moon has to go rather more than once round the earth, because the earth has not been standing still, but by the time the moon has got once absolutely or sidereally round the earth we and the moon together have gone forward 43,250,000 miles in our joint annual journey round the sun. And as the moon goes round the earth in the same direction as the earth round the sun, or what we call from west to east, she has to travel farther round the earth to get again into a line between it and the sun. If the earth stood still, the solar period of the moon would be only 27d. 7h. 43m. 11·5s., or 27·32166 days, or 655·72 hours, which is its *sidereal* or absolute period. The other, of 29·5306 days, is called the *synodical* period, or simply, a *lunation*.

The mean synodical period of two bodies revolving round a third, really or apparently, is the time of their all three coming a second time into the same relative position. If the two go the same way, their relative or synodical velocity is evidently the difference of their separate velocities: if they go opposite ways, the sum of them; and velocities of revolution evidently vary inversely as the periods, or directly as the *reciprocals* of the periods; for 1 divided by anything is called its

reciprocal. Therefore the reciprocal of the synodical period is the difference, or the sum, of the reciprocals of the two absolute or sidereal periods. From which it follows by a common sum in fractions that the synodical period of two bodies going the same way, like the sun and moon, is the product of their several sidereal periods divided by the difference; or by the sum of them if they go opposite ways, like the moon and her nodes.

Twelve lunations, or 354·367 days, are called a *lunar year*. Therefore the moon is nearly 11 days older (until the excess is more than 29½) at the beginning of every successive year on the average. The moon's age at that time is called the *epact* of the year.

A lunation is rather shorter in summer than winter, because the sun being farthest off in summer, and his angular or apparent velocity being inversely as the square of his distance (as we shall see afterwards), he goes 60′ a day when nearest and only 58′ when farthest off. The 2·21 days' excess of a lunation over a sidereal period of the moon is due to the advance of the sun in a month; and therefore that excess is least and a lunation shortest when the sun advances least. The extreme difference of lunations is 3h. 50m.

Since the moon goes 360° round the earth in 27·322 days, she goes 13° 10½′ a day on the average; and the sun 59′. Therefore the moon advances 12° 11½′ more than the sun daily. And as the earth turns in the same direction, it has to turn that 12° 11½′ more than once round for the moon to cross the same meridian again; which makes the *lunar day* 24h. 49m.; or the time of moon-rise, or of moon-culmination, or of moon-setting, 49 minutes a day later, on the average.

Harvest Moon.—But the time of moon-rise is by no means uniformly 49 minutes later every day. In fact she rises nearly at the same time for several days in every month, in high latitudes like this. To understand this you had better take a globe, and elevate the north pole about 52° above the north horizon. If you stick two or three wafers on the ecliptic near ♈, about 12° apart, they will nearly represent the moons of successive nights, and you will see that they rise nearly at the same time, as you turn the globe from east to west while ♈ is at the east side of the horizon. For though the earth turns the other way, celestial objects turn that way relatively to the horizon. (The position of the ecliptic on terrestrial globes is of course arbitrary, provided it touches both the tropics somewhere.) Some kind of moon rises at ♈ every month, because she goes through all the signs in a month: but only one *full* moon in the year, viz., that one which comes nearest to ♈, which is when the sun is at the opposite node ♎, or the autumnal equinox. So the moon rises about full near sunset for several nights together always within a fortnight of September 23; and that is called the harvest moon. Or there may be two, equidistant from the equinox.

This uniformity of time of moon-rise for a few days in every month is also greatest when the moon's ascending node coincides with ♈; for then the moon's orbit is 28½° inclined to the equator, and still nearer to the *colatitude* of England, which is about 38°. On the other hand, when the moon's ascending node coincides with the sun's descending node ♎, the inclination of the moon's orbit to the equator is only 18½°,

and therefore the 'harvest moon' effect is considerably less. Conversely, near the spring equinox the daily difference of the time of rising of the full moon greatly exceeds the average 49 minutes, varying in like manner with the position of the nodes. In like manner the time of sun-rise varies less at the vernal equinox than at the autumnal, and that of sunset more.

Moonlight in winter.—There is much more moonlight in winter than in summer. I do not mean merely that there are longer nights for the moon to shine in; but that in our winter the brightest fortnight of the moon comes when the northern hemisphere of the earth is in the best condition for it, being turned towards the full moon, and so having it highest and longest above the horizon. You will see that in a minute, if you remember that the sun is lowest in winter, because the north pole then leans away from him; and as the full moon is opposite to the sun, the north pole must lean towards the full moon when it leans away from the sun. Therefore each hemisphere has as much longer full-moonlight than darkness in winter as it has longer sunlight than darkness in summer. The nights of short moon in winter are also the nights of new moon, when there is the least moon to lose. And the contrary of all this holds in summer, when the moon is less wanted.

In consequence of the moon's orbit being 5° inclined to the ecliptic or sun's apparent orbit, the moon may be 5° higher above our horizon than the sun ever is; or it may be so much lower. I remember a woman saying that the night when she saw a man stealing some fowls was the brightest she had ever seen. I had

the curiousity to look whether there was any real ground for the remark and was surprised to find there was; for it happened to be the night of full moon at Christmas, and just at the time (which comes once in nearly 19 years) when the moon's ascending node coincided with the sun's ascending node ♈, and so the moon really was the highest that she ever is in this latitude, viz.: $38\frac{1}{2}°$ for the *colatitude* (or difference between 90° and latitude) + $23\frac{1}{2}°$ for the inclination of the equator to ecliptic + 5° for the moon's extreme elevation above the ecliptic, = 67°, or three quarters of the height from the horizon to the *zenith*, or highest point of the heavens. Whether she was also near perigee I do not know.

Azimuth of Sun and Moon.—Most people suppose that the sun and moon are due east and west 6 hours from their culmination on the meridian, i.e. at 6 o'clock always for the sun in the summer half-year, and at variable times for the moon. But in fact they never are, except on the single days when they cross the equator. We will confine ourselves to the sun for simplicity, and because his place is much more noticed than the moon's except by astronomers. At midsummer in this latitude (say 52°), so far from the sun being east and west at 6 o'clock, he is so at 7:20 A.M. and 4:40 P.M.; and at 6:48 A.M. and 5:12 P.M. on 1 May and 8 August; and at 6:32 A.M. and 5:28 P.M. on 16 April and 27 August. Farther south the deviation is still greater: in the latitude of Rome the sun is east at 8 A.M. at midsummer, though he does not rise till 4, which is rather later than he rises here. He not only rises and sets farther north at any given place as

the days lengthen, but is farther north besides at any given time from noon. This can only be calculated by spherical trigonometry, but you can find the sun or moon's horizontal distance from E. and W. (which is called their *azimuth*) at any time, or their time of being there, by a terrestrial globe as follows:

Elevate the globe for this latitude, i.e. set the pole 52° above the north horizon. The brazen meridian is generally graduated from the pole in one direction and the equator in the other. The sun or moon's *declination* is like terrestrial latitude, reckoned from the equator northward, and is given for them both daily in Whitaker's Almanac. Mark any spot on the globe, at the proper distance from the equator for the declination, and lay that against the meridian, and turn the hour-circle to XII. To find the time of the sun being due east, lay a long strip of stiff paper or thin brass over the globe from E. to W. of the horizon and through the *zenith* (38° from the pole, as the equator is 38° from the horizon). Bring the sun to that vertical great circle (which is called the *prime vertical* when it goes through E. and W.). The number of the hour circle then on the meridian is the hour from noon at which the sun is E. or W. Or conversely, if you want his azimuth at any hour, turn the globe and hour circle so much from the meridian, and keep it steady there while you put the vertical great circle through the zenith and the sun, and see where it cuts the horizon, which is graduated from E. and W., and that is the sun's azimuth. If your great circle is graduated, you will also see at once the *altitude* above the horizon. Many globes have a loose quadrant of brass so gra-

duated, and capable of being screwed to the meridian at the zenith, so as to turn on a pivot azimuthally, i.e., every point in it moving parallel to the horizon.

ECLIPSES.

An eclipse of the moon is the moon passing through the shadow of the earth from the sun, which darkens the moon, as the shadow of a tree darkens the ground, only much more, because there is nothing to reflect and spread the light over the moon in shadow (subject to what is said at p. 152), as sunlight is spread by the air in some way that no other light is. You know that the shadow of the tree stands still, and is equally visible to you wherever you stand, if you are near enough to see it at all. And so you never read in the almanac that there is going to be an eclipse of the moon visible in Spain, or Norway, but not here; except that of course it is only to be seen by the people on that side of the earth which faces the moon just then.

But eclipses of the sun are always announced to be visible at certain places, and astronomers travel into distant countries to see total eclipses of the sun; because that is the only time when the corona and other envelopes of the sun can be seen directly, or his atmosphere investigated at all, as described at p. 93. If you put a small screen between you and the fire, it will only hide the fire while you are in one place: that is, the eclipse of the fire is only visible to you there, and if you move to another part of the room you see no eclipse. So the new moon may hide the sun from America while we can see him here. And that is why there are more eclipses of the

moon than of the sun in any one country, though there are in the whole world more eclipses of the sun.

But now comes the question, why is there not an eclipse of the sun and moon at every new and full moon? If the moon were always in the plane of the orbit of the sun or earth (whichever we like to call it) there would be, and that is why that plane is called the ecliptic. But she is not: the sun and earth and moon could not be represented by balls floating on the same water. The moon's orbit is inclined to the ecliptic 5° 9′; and therefore the moon is only in the ecliptic, or on a level with the earth and sun, twice in each lunation, when she is just rising above or going below the ecliptic at either of her nodes; and therefore eclipses can only happen when the new or full moon are near a node.

Moreover the nodes do not stay in the same place, but keep moving backwards, so that each node is nearly 47′ less than half a circle or 180° from the previous one; or they recede 19° 21′ a year; or the line of nodes revolves in 6793·39 days or 18y. 7m. 6d. This twisting motion of the plane of the moon's orbit backwards is quite distinct from the revolution of the apsides of the orbit forwards in about half the time. The recession of the nodes might be represented by dipping a sheet of tin into the water inclined 5° 9′ to it, and twisting it round from left to right, always keeping the same inclination; while the advance of the apsides would be represented by an elliptical plate fixed loosely to the tin sheet by a pin through the focus, and turned from right to left, keeping the focus on the water.

Unless a new moon happens within 17° of a node, the

sun escapes being eclipsed to any part of the earth; and unless a full moon happens within 11° 21′ of a node, it clears the earth's shadow and is not eclipsed: passing through the penumbra only is not reckoned an eclipse of the moon. What the penumbra is will be explained presently. These limits are for mean distances, and are rather wider when the moon is nearer or the sun farther. But the line of nodes must be crossed by the sun in his motion round the earth twice a year, and so there are at least two opportunities for an eclipse of each kind; and there may be more. There are never less than two eclipses of the sun every year, and there can easily be four partial ones. For there are 34° at each node within which one can happen; and if the moon gives the sun one partial eclipse 17° before reaching the node, she will be in time to give another at the next new moon, as the sun will only have moved about 29° in that time. Indeed there can be five solar eclipses, as there can be 13 new moons in a year, or 13 full moons, though not 13 of both. Therefore if there is an eclipse, lunar or solar, before January 11, there may be a similar one at the end of December, if the position of the nodes is favourable, which it is the more likely to be from their receding, so that the sun reaches the same node again in 346·6 days,* or rather less than 12 lunations (354 days). There can only be three lunar eclipses in the year, at intervals of 177 days or 6 lunations, as the moon can only be eclipsed once within the 22° at each node, and she may escape altogether. So there are never more than 7 eclipses

* This synodical period is calculated from the sidereal year and the 6793·4 days of nodal revolution, by the rule at p. 138.

in a year, for if there are 5 solar there cannot be 3 lunar, and *vice versâ.*

If you put two candles close together, and hold a stick between them and the wall at a suitable distance, it will cast a narrow black shadow on the wall, bounded by two paler ones. Within the black shadow both candles are eclipsed, but in the pale shadows only one. A lamp with a very wide flame would represent the sun more accurately, but two candles are easier to manage. When the sun is narrowed by an eclipse the fringes of shadows are narrower than usual. So the earth in a lunar eclipse casts a black shadow on the moon, called the *umbra*, within which the sun is totally eclipsed to the moon, surrounded by a *penumbra*, which is a pale shadow on the moon to us, and would be a partial eclipse of the sun to those parts of the moon which the penumbra covers. In like manner the moon hides the whole sun from the earth wherever the umbra or full shadow of the moon falls in a solar eclipse; and those parts of the earth which see the sun partially eclipsed would appear covered with a pale shadow to people in the moon. The shadow of the earth's penumbra on the moon is so slight that it is hardly noticed.

Solar eclipses.—The sun's diameter being 396 times the moon's, her umbra is a cone which runs to a point at a distance = a 395th of the sun's distance, or 229,000 miles beyond the moon when the sun is nearest, and nearly 237,000 when he is farthest off; and therefore falls short of the earth's surface if the new moon is near apogee, and reaches beyond it at perigee, and almost exactly to it at mean distances of the sun and

moon (p. 117). When the umbra does not reach the earth it is only touched by the penumbra, and the parts of the earth which are swept over by the middle of the penumbra then see a partial but annular eclipse of the sun; for an annular eclipse is only a particular kind of partial one. Of course every eclipse begins and ends as a partial one.* The places a little way off that middle line see a partial but not an annular eclipse. If the moon is near enough for the umbra to reach the earth there is a total eclipse of the sun at all the places which it covers. It can only cover a spot 148 miles in diameter when the moon is nearest to the earth, and the sun farthest, or the total eclipse can only move over a zone of that width, and the *totality* never lasts above 7 minutes at one place, and very seldom so much.

For a total eclipse can only last as long at one place as that place takes to move through 148 miles by the two motions of the earth, bearing in mind the motion of the moon and her shadow at the same time. As the moon's synodical period with the sun is 708·734 hours we may consider the sun and moon and shadow at rest, and the earth going round the moon in that time in an orbit of 238,820 miles mean radius (p. 116) and therefore 2122 miles an hour. But we want the velocity when she is in perigee, for it is only then that there

* Some pictures of eclipses in books are likely to give an erroneous idea of an annular eclipse, by making the middle of the penumbra black like the umbra, which is contrary to the fact. No picture seems to me likely to make the matter clearer: in fact I had drawn one, and written a description of eclipses to suit it; but it was longer than this instead of shorter, and harder rather than easier, so far as I could judge. There is a good account of the principal 'solar eclipses' under that head in the English Cyclopædia.

can be a long total eclipse. By one of Kepler's laws which we shall consider hereafter, the moon's (or supposed earth's) velocity × distance at either perigee or apogee = the mean velocity × mean distance; and as the distance at perigee may be as little as 221,600 miles, it follows that the velocity will then be 2287 miles an hour or 36 a minute. But a place not far from the equator is carried by rotation about 16 miles a minute in the same direction as the moon's shadow goes, and so such a place will move through the shadow about 20 miles a minute; and therefore under a combination of the most favourable conditions the totality may last something more than 7 minutes. But of course that happens very seldom indeed. The eclipse of 17 August 1868 lasted a little less than 7 minutes in India, and is the longest on record. But a total eclipse may take 3½ hours to pass over the whole earth centrally.

It may also be shown, but at too great length to introduce here, that in the long run there will be rather less than 4 total eclipses in 9 years, visible somewhere on the earth.

Such different accounts have been given of the darkness in total eclipses, that we can only conclude that the old ones were much exaggerated by the people's alarm at such an unusual kind of darkness as all observers agree that it is. It is also accompanied with cold, as is natural, and the darkness of the shortest total eclipse is said to be immeasurably greater than that of the largest partial or annular one. In a total eclipse the moon is more visible than the 'old moon in the new moon's arms' by reflection from the earth,

because there is not even a bright streak of directly illuminated moon to be contrasted with it.

Mr. Hind, the editor of the Nautical Almanac, published in the *Times* of 28 July 1871 and 19 July 1872 accounts of some old total eclipses; and it is remarkable that London has never seen but one, on 3 May 1715, though there was another in the midland counties in 1140, when it is said people lighted candles at noon, and another in the south-western counties on 22 May 1724, and two in Scotland in 1433 and 1598. He predicts that London will see another on 11 August 1999. The most recent corrections of the moon's motion, and the earth's with it, have enabled the dates of certain much more ancient eclipses to be identified with the events narrated with them. But there is one, the great eclipse of Xerxes, which still resists all explanation, unless the date of the battle of Salamis, which shortly followed it, is to be altered from B.C. 480 to 478, when Mr. Hind finds there was a very large eclipse at Sardis, though not quite total, on February 17. Herodotus (vii. 37) thus describes it: 'When the army of the Persians was setting out from 'Sardis to Abydos in spring, the sun leaving his seat in 'heaven became invisible in a perfectly clear sky, with-'out clouds, and instead of day it became night' (exactly the phrase which he used for the eclipse of 485, predicted by Thales). 'Xerxes seeing this became uneasy and 'inquired of his magi what this phænomenon might 'mean. They answered that it signified to the Greeks 'the destruction of their cities, because the sun was the '*significator* (in astrological language) of Greece, but 'the moon of them'—the Persians.

This was written by the 'father of history' of a very momentous event almost (if not quite) within his own lifetime: the invasion of Greece I mean, not merely the eclipse; and a stronger description of a solar eclipse in every word of it could not be written. Yet the Astronomer Royal, in the R. A. S. 'Memoirs' of 1853, before Adams's great correction of the lunar theory, actually propounded, as an 'extremely probable' solution of the difficulty, that Herodotus was mistaken, and wrote all this of an eclipse of the *moon*, not in 480 but in 479: a notable specimen of the modern fashion of correcting almost contemporaneous history by conjectures and supposed probabilities.* And as a further commentary on it, he has since concluded, on other grounds which will be noticed afterwards, that 'there is still some serious defect in the lunar theory.' What historians may say to Mr. Hind's necessity for altering the date of Salamis 2 years, and therefore of Marathon which was 10 years ~~after~~ it, I cannot tell. If those dates are absolutely immoveable on historical grounds, there must be something very wrong in the lunar theory. If so, it is strange that the calculations should agree with the received dates of the eclipses of Thales in 485, of Agathocles in 310, and the still more valuable one of Nineveh on 15 June 763, which it seems is recorded with its date in one of the Nineveh tablets in the British Museum. But this also, according to Mr. Hind, was not quite total; though he

* After this it is not surprising to find him publishing a collection of equally bold and positive conjectural corrections of the Old Testament, and of the received authorship of various parts of it, and even of the characters and objects of many of the principal persons in it.

suggests that a very slight correction of the lunar elements would make it so. The same may possibly be true of the eclipse of 478.

Lunar eclipses are of much less interest to astronomers than solar, and require little further explanation. The earth is so large that its umbra in a lunar eclipse reaches far beyond the moon, and may be as much as 5950 miles wide where the moon crosses it, and is never less than 5650. And as the moon is only 2160 there can never be anything like an annular eclipse of the moon. A total eclipse may last 1h. 45m., and it is an hour more than that from the first contact to the last, since the moon takes about an hour to move across her own width. The earth's penumbra is 10,200 miles wide at the mean distance of the moon. These things cannot be calculated without geometry, so I only give the results. I have already given her mean velocity of passing through the shadow—the same as the earth's in a solar eclipse, and the maximum: the minimum is nearly 2000 miles an hour. The moon goes apparently as well as really eastward through the shadow or over the sun, because she goes round the earth 13·37 times faster than the sun or the shadow, and the earth's rotation eastward only makes them both appear to move westward equally.

The moon is very seldom so totally eclipsed as to be invisible. A lunar eclipse differs from a solar one in the boundaries of the umbra and penumbra being much less clearly marked. We either positively see or do not see the whole or part of the sun, and the part which is not eclipsed remains as bright as usual. For a solar eclipse is simplified by the moon having no

atmosphere to refract the sun's rays as they pass by her. But the penumbra of the earth is only a gradually darkening shadow, into which the moon cannot be seen definitely to enter. Moreover the umbra itself is invaded and coloured red by rays which would pass by the earth and beyond the umbra altogether if they were not refracted and turned inwards by our atmosphere, which they have to pass through as obliquely as possible in skirting the edges of the earth. And they are bent inwards so much that they cover the whole umbra at the distance where the moon crosses it, leaving the undiluted umbra a shorter cone which does not reach the moon. In other words, there is no time when she is quite invisible and does not receive some rays of the sun through the refraction of the earth's atmosphere; except that the rays are sometimes stopped instead of refracted by the earth's atmosphere being cloudy where they pass, and then the eclipsed moon is quite invisible, and stars passing behind her disappear suddenly without any visible cause.

CYCLES OF THE MOON, AND EASTER.

It was discovered long ago, and was once thought important, that 600 equinoctial years = 7421 lunations within a few hours. But a cycle of that length is of no practical value, especially as it does not also coincide with any number of revolutions or half revolutions of the nodes.

The Saros.—But there are two other lunar cycles of much more useful length, and one of them has still an important bearing on our calendar. They are so

nearly of the same length, that they are liable to be confounded, though they have no relation to each other. The first is the Chaldæan Saros (which means restitution) or cycle of eclipses. 223 lunations are 6585·324 days, or 18 of our years + 10 or 11 days (according as they include 5 or 4 leap years) and 7h. 41m. That is only 45m. short of 242 nodical months (p. 130); and, by another remarkable coincidence, it is only 5 hours short of 239 anomalistic months (p. 128). Consequently the sun and moon and her nodes will then be all again in the same relative position, and their distances from the earth also practically the same; for the 11 days and the 5 hours respectively make no such difference in their distances from perigee as to affect their distances from the earth sensibly. Consequently all the eclipses will recur in the same order and magnitude after this period: but not at the same times of day, on account of the 7h. 41m. difference, and so they may not be visible at the same places. Therefore the Chaldæans, who had ascertained all this somehow, made a more complete saros of three such periods, = 54 years + 32 or 33 days within an hour. It is singular that even the stars are very nearly in the same position relatively to the sun and moon at the end of a saros, for it only exceeds 241 sidereal lunations (p. 137) by 19 hours.*

The Metonic cycle is so called because it was discovered by Meton an Athenian, B.C. 433. It has

* See the Appendix on Chaldæan Astronomy in Mr. Proctor's 'Saturn,' p. 173. Sir J. Herschel does not notice this anomalistic coincidence, which affects the magnitude and even the nature of a solar eclipse, making it perhaps either annular or total.

nothing to do with eclipses, but it is of far more importance than the Saros, as the time of Easter has been fixed by it as long as it has been fixed by any rule at all; for many centuries without any modification, and latterly with some; but still that cycle is the rule. 235 lunations take 6939·69 days, or so little less than 19 years of $365\frac{1}{4}$ days that they only differ by a day in 322 years. There is indeed a whole day's difference according as the 19 years include 4 or 5 leap years; and the complete cycle is 4×19 or 76 years. But the world has always been content to use only 19 *patterns* of years at once for finding the Easter moon: only they are now shifted in the calendar 7 times in 1200 years, as you will see at p. 159. Neglecting for the present that difference between leap and common years, which corrects itself every 4 years, and the moon's coming a day sooner in 322 years, we may say that the new and full moons come again on the same days of the same months every 19th year. Therefore there are only 19 out of the 30 days after the vernal equinox on which the equinoctial full moon can fall; and the 19 *golden numbers* prefixed to those days in the Prayer-book calendar mean that the day against the golden number of the year is the day of Paschal moon, or full moon next before Easter Sunday.

For by the rule which has existed over all the world since the first Council of Nice in 325, Easter is the Sunday after the full moon next after the 20th of March. The Council left the moon to be found as it might be; and further disputes arose on that; which were ended by Pope Hilarius in 463 ordaining what has ever since been the law of church and state, that

the Paschal moon should not be the actual full moon to be found by astronomers, but the 14th day of the moon by the Metonic cycle, though the real full moon is generally on the 15th day from new. This was done to prevent Easter falling on the Jewish Passover.

Consequently the Paschal moon often differs from the true equinoctial moon by a day or two, and Easter may be a week, or even five weeks earlier or later than it would be if it followed the real moon. Indeed unless an 'Easter meridian' were agreed on for the whole world, it might still differ five weeks in different places in the same country, even if it were fixed by astronomers and therefore made incalculable either forwards or backwards by anybody else. For if there is full moon in London very early in the morning of Saturday March 21, Easter would be the next day there: but that same full moon may be on Friday night of March 20 at Exeter or Oxford by true time, and therefore would not be the Paschal moon there, which would be on Sunday April 19, and Easter not till April 26: which it never is now.

The correction of the Metonic cycle for the purpose of keeping the ecclesiastical full moon tolerably near the real one on the average involves the whole subject of the reformation of the calendar, or the change from Old to New Style, which is intimately connected with astronomy. It occupies many pages in the books which treat of it fully, but I hope to explain all that is important of it in a short compass. The most complete treatise on it is De Morgan's in the 'Companion to the Almanac' for 1845 and 1846, and his article on Easter in the English Cyclopædia. There is also a

very clear paper about it in the Philosophical Transactions of 1750 by the Lord Macclesfield of that time, which prepared the way for the Act of Parliament, 24 G. II., cap. 23 (1751), for changing the style in September 1752.* But none of these contain a table for old style as well as new, such as you will find at page 164: nor do the Prayer-books, as they only have the calendar and rules of that Act for all years after 1599. The Paschal full moon is in fact a Parliamentary phænomenon, not an astronomical one. In order to explain it we must consider generally—

The Calendar.—It was known very early that a year is no exact number of days; and different nations had different plans for occasionally adding or *intercalating* days, to make up for the fraction lost in each year. It is not worth while to go into the history of these contrivances. The present scheme of three years of 365 days and a fourth of 366 was invented for Julius Cæsar by Sosigenes of Alexandria B.C. 45, and lasted without alteration (except a temporary mistake corrected by Augustus) until the time of Pope Gregory XIII. It was then found that the real equinox fell ten days before the nominal one of March 21, and that Easter had got four days wrong besides, from the error in the Metonic cycle. Ten days were accordingly struck out of the calendar between the 4th and 15th of October 1582; and to prevent the error for the future

* You will find in the English and in Rees's Cyclopdiæ, a great deal also about the epact, or age of the (ecclesiastical) moon at the beginning of the year. But we can do as well without it, and the subject wants no superfluous complication. De Morgan also published a 'Book of Almanacs,' containing the 35 patterns of years for the 35 possible days of Easter, according to the rules: if they followed the real moon there would be 36 almanacs.

it was decreed that every 100th year should lose its leap day except those divisible by 400. I will describe the new Easter arrangements presently.

This is the Gregorian or New Style, which was invented by Clavius, a Jesuit, and adopted in all the Roman Catholic countries before the end of 1582, and by the Protestant German states in 1700, but not by us till 1752, and Sweden the year after, and not yet by Russia and the Greek church, which is therefore now 12 days wrong. In 1751 Parliament enacted that the day after 2 September 1752 should be September 14: dropping thus 11 days, because we had got one day more wrong by allowing 1700 to be a leap year. Of course due provision was made that nobody should lose or gain 11 days' interest on their debts; nevertheless the Act caused riots among the common people, who cried out 'Give us back our eleven days,' as if they would die so much the sooner for the loss of them. Fortunately they were not masters.

Moreover you must remember in reading old books that the years legally began on March 25 until the change of style; and therefore such an event as the execution of Charles I. on Tuesday, 30 January 1649, as we call it now, is said in old books to have been in 1648. A year of the average length of 365¼ days is called a Julian year; but the years of the *Julian era* are reckoned from 1 January 4713 B.C., an arbitrary epoch which was invented for reasons of no consequence now; so 1876 is the 6589th of the 'Julian era.' For there was no A.D. 0, and the 19th century of our era (which probably begins 4 years after the real birth of Christ) began on 1 January 1801, not 1800.

To turn Old style into New, add 11 to the day of the month in the century beginning 18 Feb. 1700, which becomes 1 March, as that February would have no leap day by New style: add 10 days in the *two* centuries after 19 Feb. 1500; 9 days in the century beginning 20 Feb. 1400, and so on.

Now let us see what amount of error the Gregorian calendar still leaves, after professing to correct itself every 400 years. A million Julian years are 365,250,000 days; but a million equinoctial years are 365,242,216 days. Therefore the problem is how to drop 7784 leap days in a million years in some neat and simple way: that is, one day in 128·47 years. But the Gregorian plan drops a day in 133·333 years in the long run. Therefore the error is 4·86 days in 128·47 × 133·333 years, or a day in 3524 years. Accordingly Sir J. Herschel proposed to carry the correction a step farther by making every 4000th year lose its leap day, though it is divisible by 400: which would go for about 28,000 years without an error of a day.

But I think the mere statement of the problem suggests the best solution of it: $128 = 32 \times 4$; and that is a number easy to remember, being formed by 7 successive duplications of 1, or 2^7. Therefore the simplest of all plans would be just to let every 128th year lose its leap day; and that only makes a day wrong in 35,440 years: which is ten times more accurate than the Gregorian scheme, and seven times better than another which is commended by Sir J. Herschel, for dropping one day in 132 years (misprinted or hastily written in his 'Outlines' 128) by postponing the leap day of every 32nd year to the 33rd; which would also be most

inconvenient, by making the leap years no longer all divisible by 4.

Even if reckoning by centuries is thought essential, still the Gregorian scheme is not the best. It would be both more correct and more symmetrical to drop the leap day of every century except the fifth, instead of the fourth.* For that makes leap years of 2500, 3000, &c., instead of 2400, 2800, 3200, and only accumulates an error of a day in 4646 years: which again could be rectified in 5000 years more completely than the Gregorian error in 4000.

Correction for Easter.—The next question was how Easter was to be set right, and the Metonic cycle modified for the future, to prevent the ecclesiastical and real moons from getting wider and wider apart. Under the Old style the golden numbers were never shifted; for it assumed the Metonic cycle to be perfect; and therefore the same cycle of almanacs recurred every $7 \times 4 \times 19$ or 532 years. For the future they wanted a new set of pattern years, to be indicated by the 19 golden numbers, with a contrivance for shifting them again when they had got a whole day wrong. The golden numbers of successive years have always run regularly from 1 to 19 without any dislocation. So the first thing was to put them against a new set of days between March 21 and April 18 (inclusive), and that arrangement was to last till the year 1700. Then they are to be shifted on this plan: at every century

* Friar Bacon in 1267 urged Pope Clement IV. to reform the calendar on a plan equivalent to this. He had found out somehow the length of the year more exactly than anybody else then or for a long time afterwards. But he was put in prison for ten years instead; which has been merged in the greater fame of Galileo's imprisonment.

divisible by neither 3 nor 4 they are all to be advanced a day in the calendar, and to be set back a day at every century divisible both by 3 and 4, i.e. by 12, and not altered in the others.

The reason of that contrivance was this. Suppose the sun and moon to be right at some century divisible by 12, which we may call 0 for this purpose. Then at the year 100 the Gregorian year loses a (leap) day, and so the moons of the next century will all come a day later than if that day had not been dropped, and the golden numbers must be advanced a day accordingly; and the same at 200. At 300 the year again loses a day; but by that time the Metonic cycle has put the nominal moon nearly a day too forward, and therefore ought to lose a day; and so they balance without any alteration. In 400 the year does not lose a day, and the moon does not, and so again they are right; and thus you may go on till 1200, which drops no day; but the moon has then lost a day since 900, and so the golden numbers have to be put back. But Clavius supposed the moon to lose a day by the Metonic cycle in 300 years more exactly than it does.

You may see the result of all this in the Prayer-book in II. and III. of the 'General tables for finding the Sunday letter and the places of the golden numbers,' for as long as the world and the Gregorian rules may last. The smaller numbers by the side of the centuries in Table II. show at once how many days the golden numbers are advanced on the whole since 1600, which will evidently be 5 days in 1200 years on the average, being set back once in that time, and forward 9 times.

Now let us see how nearly this keeps Easter right, assuming it to have been set right in 1600. Though 235 lunations take 1h. 26m. less than 19 Julian years, they take 2h. 6m. *more* than 19 equinoctial years, so that the moons of every 19th year, or of every year with the same golden number, come a day later after 216 years. In other words, if our years were correctly adjusted, by dropping two days in 257 years, a new or full moon of March 21 now would fall on March 22 in any year of the same golden number after 216 years, and on March 23 after 432 years, and so on. So that instead of the golden numbers advancing at the rate of 5 days in 1200 years, they ought to advance 5 days in 1080 years. Again 235 lunations exceed 19 Gregorian years (whose average length is 365·2425 days) so much that the moon advances a day in 232 of those years, or 5 days in 1160 and not 1200 years: so that the Gregorian rule does not keep the moon right even for the average Gregorian year, which is itself wrong.

The result of all this is that the rules for keeping Easter neither keep it by the real equinoctial moon of each year, nor by a moon which is right on the average of a long period, either for the real equinox, or for the artificial year which we adopt, and sometimes leave it five weeks off the real time. It seems that even Clavius the Jesuit, who did the astronomical work of reforming the calendar for Gregory XIII., ventured to publish the suggestion that it would be better to keep Easter by the sun, making it the Sunday after some given day, instead of letting all the great festivals and holidays, except Christmas, wander over five weeks of the calendar in the vain attempt to follow the moon.

perhaps before the year 2000 the world will be in a condition to revise the calendar and reconsider that question without prejudice.*

To find the days of the Week.—But whatever is to be done hereafter, Easter and the days of the week for any given days of the month can only be found for the past and the present by the existing rules, and so it is important to understand them. The table in the Prayer-book for finding the Sunday letter, or the day of the week for any given day of the month, is wrong for any year before September 1752. They copied the Gregorian tables into the Act of 1751 without taking the trouble to adapt them to this country, or to say that they only applied to the new style; and therefore that table will give wrong days of the week backwards before 1752. The proper table and rule are these:

0	A	1800 to 1899 inc.	For countries which changed their style in 1582 it must be,
1	G	14 Sep. 1752 to 1799.	1 G 1700.
2	F	2500.	2 F 1583 to 1699: the rest as before.
3	E	2400, 2300.	
4	D	2200.	
5	C	2100, and every year of old style.	
6	B	2000, 1900.	

Rule for finding the Sunday letter.—Add to the year its fourth part, omitting fractions, and also the number set opposite to it or to the last century before it in this table, and then divide by 7; the remainder

* The equinoctial full moon appears to have some effect on the weather. The Easter fortnight has only once passed without frost or snow in England since 1837 certainly. Before that I did not observe it. The coldest Easter I ever saw was almost the latest possible.

over indicates the Sunday letter. But until the end of February in leap years the letter above the one so indicated is the Sunday letter. When that is found you have only to look at the Prayer-book calendar, where all the days marked with that letter are Sundays in that year. The reason of the rule is that a common year ends on the same day of the week as it begins, or has one day over the 52 weeks, and a leap year two days. Therefore starting from 0, there are as many days over beyond some entire weeks as the number of the year + the number of leap years, and as many days of the week over as the remainder after dividing those days by 7. The arrangement of the letters in the calendar, beginning with A for January 1, is only arbitrary, and happens to require the constant addendum of 5 to make the Sundays of all the years until the change of style come right.* After 1752 the dropping of leap day in every century not divisible by 4 displaces the Sunday letters of the next 100 years by one day.

To find Easter.—Either of the tables in the Prayer-book for this purpose is right since 1752. I have made a similar one for old style, which is right for every country until it changed, and therefore here till 1753; for Easter of 1752 went by old style. First come all the possible days of the Paschal moon, and therefore of the golden numbers, and a week more to reach a Sunday when that moon is on Sunday April 18. Then come the Sunday letters of those days

* You may test this by the date above given for the death of Charles I., or by the older date of Columbus's sailing for America (which sailors appear to have forgotten was not an unlucky journey certainly) on *Friday*, 3 August 1492 (Helps's 'Columbus').

Possible days of Full Moon and Easter.		O. S.	1753 to 1899.	1900 to 2199	1583 to 1699.
March 21	C	16	14	..	3
22	D	5	3	14	..
23	E	..	..	3	11
24	F	13	11	..	..
25	G	2	..	11	19
26	A	..	19	..	8
27	B	10	8	19	..
28	C	..	..	8	16
29	D	18	16	..	5
30	E	7	5	16	..
31	F	..	..	5	13
April 1	G	15	13	..	2
2	A	4	2	13	..
3	B	..	..	2	10
4	C	12	10	..	..
5	D	1	..	10	18
6	E	..	18	..	7
7	F	9	7	18	..
8	G	..	..	7	15
9	A	17	15	..	4
10	B	6	4	15	..
11	C	..	..	4	12
12	D	14	12	..	1
13	E	3	1	12	..
14	F	..	..	1	9
15	G	11	9	..	..
16	A	..	..	9	17
17	B	19	17	17	6
18	C	8	6	6	14
19	D				
20	E				
21	F				
22	G				
23	A				
24	B				
25	C				
		Old Style everywhere.	1700 to 1899 Rome. 1753 to 1899 England.	English and Roman.	Roman only.

Sunday Letter of the first day of each month in the Calendar.	
G	April. July. Jan. 7
F	Sept. Dec. Jan. 6.
E	June. Jan. 5.
D	Feb. March. Nov.
C	August. Jan. 3.
B	May. Jan. 2.
A	January. October.

according to the calendar. The column headed O. S. has the golden numbers as they stood until the change of style. The next has them up to 1899 (inclusive) from 1753 in England, and from 1700 in Roman Catholic countries; and the following one from 1900 to 2199, unless the rule is altered before then. The column headed 1583 to 1699 is only for the countries which changed in 1582 (see English Cyc., Easter).

The way to use the table is this. Add 1 to the year and divide by 19, and the remainder is the golden number, no remainder corresponding to 19. Find that in the column which the year belongs to, and run your eye horizontally back to the day of the month opposite to it, which is the Paschal moon, or the 14th day of the moon according to the Gregorian rules. The Sunday after it is found by the rule which I gave for the Sunday letter, and that is Easter Sunday.

THE TIDES.

The rising of the tides as much later every day as the moon is later in coming to the meridian must have been the first thing that suggested the idea that they are due in some way to the moon's attraction. Their connection with the sun might be less evident, though thoughtful men must have perceived it from the fact that the tides are greatest at new and full moons and least at half moons. Still it was not till Newton came that any real theory of the tides was propounded, and even he left it imperfect. His theory is very simple.

Every particle of water on the earth is attracted towards the moon (taking her first) with a force proportioned to her mass and the inverse square of her distance from that particle. Therefore she attracts the water on the near side of the earth more than she attracts the earth (at its centre), and the water on the far side less; which comes to the same thing for tidal purposes as if the moon were cut into two nearly equal parts, and the larger half left in the real place of the moon and the smaller one put exactly opposite, leaving the earth's centre unmoved. You will see presently why the two halves must not be quite equal. The moon then diminishes the force of gravity towards the earth's centre both under her and on the opposite side, and so the water rises there from the two cross sides of the earth where gravity is not diminished, but is rather increased, as we shall see, until the increased height and mass balances the difference of gravity.

But that is not all. If you pull two balls not far apart with long strings of equal length held in one hand, you will also pull them towards each other, with a force which, you must take it as proved, varies as the angle between the strings (so long as it is a small one), i.e. as the distance of the balls apart divided by the length of the strings. In the same way the moon's attraction draws in all the waters which lie at or near 90° from the point facing her, which is the same as if the earth's attraction on them were increased. But this contractive force, or the *resolved part* of the moon's attraction on the sides of the earth, towards the centre, is only half the other separating or differential force, as

I will show you presently. Therefore if you call the contractive force 1, the other will be 2, and there is altogether a force of 3 tending to make the water facing the moon, and at the back of the earth opposite to her, higher than the water at 90° from those places.

The sun does the same in all respects, but in a less degree; for although the general attraction of the moon on the earth is very small compared with the sun's, yet her differential attraction and her contractive force on the opposite sides of the earth are greater, because she is so much nearer, and both these forces depend on the proportion of the earth's radius to the distance of the sun and moon respectively, as in the case of 'precession' (p. 73). If you like to see the calculation of the actual amount and effect of the tidal forces of the sun and moon, and the proportion which they bear to gravity, or to the earth's attraction on its own water, it can be done as follows.

We may take any length and any mass for the units of length and mass, for a foot and a pound are merely arbitrary ones convenient for measuring on a small scale. It will save trouble to call the earth's radius and mass each 1, and then the sun is 322,700, and his distance 23,213, and the moon is ·0123, and her distance 60 from the earth's centre, but 59 and 61 from the near and far sides of the earth respectively. Then the moon's attraction is to terrestrial gravity as ·0123 to 60^2; and so her attraction on the near side is $\frac{\cdot 0123 \times \text{gravity},}{59^2}$ and on the far side $\frac{\cdot 0123 \times \text{gravity}}{61^3}$

Now we want the difference between each of these and the attraction at the distance 60; and if you take the

trouble to calculate it you will find that the difference on the near side of the earth is a little more, and that on the far side a little less, than $\frac{2 \times \cdot 0123 \text{ gravity}}{60^3}$; * and that is why the moon must not be divided into two quite equal halves for tidal purposes; the tidal force under the real moon is about a 20th more than on the far side of the earth; but practically that last fraction represents both of them, and we may say that the moon's differential force at the surface of the earth or at distance 1 from its centre is to gravity as twice the moon's mass is to the cube of her distance. In like manner the sun's differential force is to gravity as twice his mass, or $2 \times 322{,}700$, is to the cube of his distance measured in earth's radii, or $23{,}213^3$. And there is no sensible difference between the sun's tidal force on the near and far sides of the earth, as the difference of distance is only an 11,606th.

And to each of these half as much more has to be added for the contractive force; for that is the general attraction of the sun or moon on the earth $\times$ the small fraction which has the earth's radius or 1 for numerator and the sun's or moon's distance for denominator. Thus the same cubes of distance come in as before, but not the 2 in the numerator; and the whole tidal force of $\frac{\text{sun}}{\text{gravity}} = \frac{322{,}700 \times 3}{23{,}213^3}$; and of $\frac{\text{moon}}{\text{gravity}} = \frac{\cdot 0123 \times 3}{60^3}$.

You will find, if you work out these figures, that

* With the earth's radius as the unit of length we must remember that gravity would have to be represented by a very much smaller figure than 32·2 as usual; for that means feet, as the usual unit of length; but we have no need to use any figure for gravity here.

gravity is nearly 6 million times the moon's tidal force and 13 million times the sun's, at mean distances; and adding the $\frac{1}{6}$ and $\frac{1}{13}$ together, the attraction of the earth on its own water is above 4 million times greater than the tidal forces of the sun and moon together.

If the earth were a fluid sphere of nearly 21 million feet radius, and of uniform density = the present average density, the tidal force at every depth would be the same 4 millionth of the central attraction, since they both vary as the distance from the centre. And as the weight of a prolate spheroid is to the sphere which it contains as their different axes, the tidal force would pull out the sphere into a spheroid whose semi-axis major exceeds the minor by $5\frac{1}{4}$ feet and lies in the *line of syzygy*, pointing to the sun and moon. But the depth of the oceans being quite insignificant compared with the earth's radius, that calculation will not apply, and the actual result is that the tidal ellipticity due to sun and moon together is only a 6 millionth, or the highest tide is only $3\frac{1}{2}$ feet above the lowest in the open sea. The earth is always treated as a sphere in tidal calculations for simplicity; or rather, the calculations are only applied to the zone including the equator: beyond that, mathematics cannot yet deal with them.

There is the same kind of excess in the force of the earth on the near side of the moon, as of the moon on the near side of the earth. But though the earth is $81\frac{1}{2}$ times heavier than the moon, the moon's radius is only the 220th of the earth's distance, and so the excess of attraction on the near side of the moon over the far side is only one 75th, and therefore

quite inadequate to account for its supposed shape (p. 131), though enough to have made her a slightly prolate spheroid when she was fluid. The whole tidal force of the earth on the moon is about 120 times that of the moon on the earth, being greater as the earth's mass and gravity on the surface exceed those of the moon, and less in the proportion of their diameters.

If you cannot follow these calculations you may accept it as proved that the tidal forces of the sun and moon are as their masses directly and the cubes of their distances inversely. And their distances vary enough to make a considerable difference in these proportions at different times. When the sun is at his nearest and the moon at her farthest, he is only 364 times farther off, and the cube of that is 48 millions; but when she is nearest and his farthest his distance is 410 times hers, of which the cube is nearly 72 millions, or just half as much more. So we have those two cubes in favour of the moon to set against 26 millions for the sun, who is so much the heavier. Therefore the sun's tidal force varies from rather more than half to rather more than a third of the moon's.

But all this might be so and yet hardly any tide would be visible, if the earth always kept the same face towards the moon, as she does to the earth. There would then be only a solar tide about a foot high, which would also move so slowly round the earth that its effects would be very different from what we see now. The ebb and flow of the tide, by which alone it is felt as a great power over the world, depends upon the earth's rotation within the water,

while part of it is held up by the tidal force. The easiest way to understand the effects of the rotation is to suppose the earth fixed, and the sun going round it from east to west in 24 hours, and the moon in 24h. 49m. The moon then drags the two opposite tidal waves after her at the rate of 1003 miles an hour at the equator, and the sun two others at the rate of 1041.

Not that the water itself moves at anything like that speed, or that much of it is carried round the earth at all, except in long periods. The thing that travels with the moon is the two alternate states of elevation and the depression of the water at 90° apart. A wave is the transmission of a state, not of a body. The water is indeed moved to the very bottom of the sea, and a good deal of it moves forward, and some back again afterwards, besides being lifted and let down again. Although the tidal wave travels westward with the relative motion of the moon, the tide itself moves towards an eastern as well as a western shore, because that is the necessary effect of the whole mass of water rising. And when the advancing water is stopped by land it can only dispose of itself by rising much higher than the 3 feet of the open sea. Waves raised by a wind stir the water to a very little depth, and not much water is carried forward in them. They 'break' on a shore because the friction of the ground stops the bottom of the water from going as fast as the top, which therefore tumbles over.

Neap and Spring tides.—The sun has his two tidal waves as well as the moon, but of less than half the size on the average; and therefore it is best to consider the tide as mainly belonging to the moon and modified

by the sun, as follows. At half moons, or *quadratures*, when the sun is 90° or 6 hours from the moon, they pull across each other, and the sun tries to make high water where the moon is making low water. The moon's tidal force being more than twice the strongest prevails, but the tide is only due to the difference of the two forces, and so rises and falls least, and that is called *neap tide*.

When the moon is past quadrature and has not reached syzygy, or the line of new and full moon, the tide is kept in advance of her by the sun; but after syzygy the tide lags behind the moon, being kept back by the sun. Consequently the tide of any place is not regularly 49 minutes later every day, as if it obeyed the moon only, but sometimes as much as an hour later and sometimes only 38 minutes. This is called the *priming* and *lagging* of the tides. When the sun and moon are in syzygy, either in conjunction or opposition, they augment each other's tidal force and produce *spring tides*, which are the sum of the lunar and solar tides, and rise the highest and fall the lowest. And these again are greatest at the equinoxes, because then the sun is on the equator and the moon must be within 5° of it, and so they are in the best position for drawing the water from the sides to the front or back of the earth. For if you wanted to pull a sluggish globe round, you would wrap a string round it at the equator and pull in the plane of the equator. And they are greater still when the new or full moon is at perigee near an equinox, which must happen about every 4½ years; and greatest of all when the moon's nodes are there too, every 9¼ years. But the actual

very high tides depend on the wind concurring with these astronomical causes; and therefore predictions of them generally fail.

Weighing the moon by the tides.—We have thus far been assuming the moon's mass to be known, and calculating the proportions of the tidal forces from it. But in fact it was just the contrary. The moon's mass was first ascertained from observations of the difference of the lunar and solar tides, i.e. of spring and neap tides at various places thus. Suppose the average spring tide anywhere is 41 feet, and the neap 15; then the lunar tide is to the solar as 41 + 15 to 41 − 15, or as 28 to 13. But the moon's tidal force is to the sun's as $\frac{\text{moon}}{\text{her dist.}^3}$ to $\frac{\text{sun}}{\text{his dist.}^3}$. The sun's mean distance is 385 times the moon's and 385^3 is about 57 millions. Therefore 57 million times the moon is to the sun as 28 to 13, which makes the sun nearly 26½ million times the moon. And the sun's mass being known to be 322,700 times the earth's by other means, that makes the earth about 82 times the moon; which is near enough for this purpose, and in fact much nearer than could really be ascertained from the tides, complicated as they are by friction and other local disturbances.

Newton from imperfect measures of the tides made the earth 40 times as heavy as the moon, and Laplace 70: Sir G. Airy called it 80 in 1856. Mr. Adams' and Mr. Stone's figure is 81·5, making the moon ·0123 of the earth, an easy figure to remember. Sir J. Herschel lowered the moon to one 88th of the earth; but the 81·5 is now generally accepted. The moon's mass is not affected by the late alteration of the mass and

distance of the sun; for the sun's tidal force remains the same as before, his mass and the cube of his distance being reduced equally (p. 85).

The top of the tidal wave however does not really point to the moon at spring tides, but 45° or 3 hours behind it in the open sea, and much more where it is obstructed by land. For the inertia and friction of the water take some time to overcome, and so the effect is always behind the cause. Spring tides are also a day or two after new and full moon, because the tidal force keeps accumulating for several days while the sun and moon are near together, and there is a greater amount of it in the four days with syzygy in the middle than in the four days before syzygy. So the hottest and coldest weather is after and not at the solstices. The more the tide is impeded by land the longer it naturally is behind the proper astronomical time: in London it is two days behind. Sometimes it has to come round islands, and is divided into two streams: consequently there are places where two tides come by roads of different lengths, and so rise and fall 4 times a day; and others where the low tide by one road neutralises the high one by the other.

In running up gradually narrowing channels it rises much higher than on the sea shore; as high as 50 feet above low water at Bristol, and in some parts of the world still more, but only about 12 feet generally on an open shore. Sometimes the tide rolls up a river which gets gradually narrower, when the wind helps it, with a face like a wall and the velocity of a railway train, upsetting everything in its way. This is called the *bore* in the Severn and Avon and some other rivers,

and the *eager* in the Humber, and it is far greater in some American and Asiatic rivers. On the other hand, when the tide has to make its way into a large sea through a narrow passage, like the Straits of Gibraltar into the Mediterranean, it is unable to produce any sensible rise and fall over such a sea.

But the tide sweeps rapidly over wide and level sands, so as to overtake and drown people sometimes, because a rise of a few inches then runs over a great area of sand; and it becomes soft under the water, like a bog, or 'quick,' because moving water lifts and carries sand and stones along with it, according to their smallness and (probably) the square of its velocity. For the weight of the stones increases as the cube of their diameter, but the surface only as the square, and the power of the stream to move them varies directly as their surface and inversely as their weight; and therefore varies inversely as their diameter, or as the cube root of the weight, among stones of the same specific gravity and general shape. A river goes on rising for some time after 'slack water,' when things cease to float upwards, because the natural flow of the river downwards balances the tidal flow upwards, but both raise the water.

The wave theory of tides.—But all this Newtonian or *statical* theory of the tides is imperfect, and indeed erroneous, because it disregards their motion round the earth, which has to be kept up by the moon and sun in addition to the mere raising of the tide. It also disregards the fact that a wave once started will go on with some velocity of its own, depending on the depth of the water and the force of gravity, until it is either

worn out by friction or accelerated or retarded by some other force. Such a wave is called a 'free' wave. The statical theory however serves very well to explain most of the phænomena, though on one important point it leads to a conclusion exactly the opposite of the truth, which is again modified by a sort of accident into apparent coincidence with the true result. The hydrodynamical or wave theory of Dr. Young is a good deal more difficult;* but it is possible to give some explanation of it, on the usual assumption that we may treat the tide as going round the earth in zones or canals parallel to the equator; though this is certainly not the case when we get far from the equator; but the calculations for high latitudes and oblique currents are beyond the present power of mathematics.

Let us see then how a wave is propagated in a canal by any force which for a moment pushes the water forward through its whole depth and width, or in any way raises it by a disturbance reaching to the bottom as the moon's attraction does. The deeper it is the more water will be lifted by a given push or displacement of any vertical column or slice of the canal; simply because more water is displaced in a deep slice than a shallow one, and the more that column will overtop its neighbour for the moment. But the raised water immediately endeavours to fall again; which it can only do by pushing the adjacent column forward

* I should have thought it altogether hopeless but for the assistance of some papers kindly sent to me soon after the publication of the fourth edition of this book by an old Cambridge friend, Mr. D. D. Heath, the senior wrangler of 1832. Not that he is responsible for the manner in which I have used them, and they involved more mathematics than I must introduce here.

and upward in the same way. And thus the elevation or wave travels on, with a velocity depending on the momentary elevation of each column above the next, or on the slope of the whole wave surface; and that you see depends on the depth. It would travel unabated through the whole length of the canal, or round and round the earth, but for friction, which would in time wear it out if not kept up by some renewal or continuance of the force which started it.

It is provable by mathematics* that the velocity^2 of the crest of the wave = the depth of the water × gravity; and expressing gravity as usual by 32·2 feet per second, that means that velocity^2 per second = depth × 32·2, all in feet. But each particle of water moves very little, and quite differently and sometimes opposite to the progress of the wave. It is always advancing while it is above its own mean level, as you will see if you consider that it is the pushing of a whole vertical stratum of particles forward which produces a wave; and again the particles are always receding while they are below their mean level. Therefore every particle describes a long ellipse—unless the water has an independent current, which may be going either way, and then we have only to suppose the ellipse travelling with it as the imaginary lunar orbit travels with the earth. The velocity of each particle backwards and forwards is to the velocity of the wave as its height above and below mean level is to the whole depth of the water; which you see is a very small proportion.

* See Mr. D. D. Heath's paper in the 'Ph. Mag.' of March 1867, or Sir G. Airy on the tides in the Encyc. Metropolitana.

It is easy to calculate from the above expression for the velocity of a 'free' wave that the sea should be $12\frac{1}{2}$ miles deep for such a wave to go round the equator at the rate of 1000 miles an hour, or the same rate as the moon, considering the earth as not rotating. But the sea is nowhere probably more than 5 miles deep through many degrees of longitude, which would give a velocity of only 600 miles even if it were that depth all over. In other words, gravity acting on a wave once started is not enough to make the tide keep pace with the moon, except in rather high latitudes, where the circumference of the earth is so much less that the velocity due to gravity might carry the wave round the earth in a day. But certainly over the great mass of the ocean the moon and sun have a good deal of work to do in keeping up the tide, even independently of friction which would soon wear it out. Let us see how they must perform the operation; and we will consider the moon alone for simplicity. Then assuming the great tide wave to exist as a prolate spheroid pointing somewhere, we have to find where that is, or where the moon must be with respect to it in order to keep it up and drag it round the earth after her.

For this purpose of increasing the natural velocity we want gravity increasing, since velocity varies as $\sqrt{}$ gravity. When the prolate spheroid points to the moon, or our two imaginary halves of the moon, gravity is not increased but diminished at the highest points. The water rises until gravity and tidal force are balanced between the sides and the front and back of the earth by the water assuming the spheroidal form. The tide must fall back a long way from that position

for gravity to be materially increased upon it. But if the tidal spheroid has its flat side towards the moon gravity is increased; for the contractive tidal force varies as the distance of the water from the earth's centre (p. 166); and the differential force, which acts against gravity under our two tidal moons, also varies as the distance of that water from the centre, which is now a minimum; therefore in both cases gravity is increased, in one directly and in the other indirectly. And high water will not stand at any intermediate place (independently of friction) for the following reason.

Neither the differential nor the contractive forces draw the water either way at the four cardinal points at any moment, calling the one facing the moon S. and the others N.E.W. according to their relative positions, the moon going in the direction E.S.W.N. relatively to the earth's rotation faster the other way. But the two forces together produce a tangential force towards S. which is greatest near the half quarters S.E. and S.W.; and on the back side of the earth, where the other imaginary half of the moon is, there is a similar pair of tangential forces towards N., greatest at N.E. and N.W. Therefore no head of a wave can lie at any of those intermediate places, because it would be immediately pulled forward by the tangential force, which is there unbalanced by any other, since gravity at the top of a wave does not urge the particles one way more than the other. Consequently the prolate spheroid must not only fall a long way back from the moon as I said, but must fall back the whole 90°, and lie at E.W.—but for friction, which we will consider presently. Thus we see that the true theory of the

tides leads to the apparent paradox that the moon's attraction—on the tide in motion—produces not high water under the moon but low. And this further odd looking result follows; that the water, though not the wave, under the moon (and opposite) is always flowing backwards: for we saw that the water below the mean level of an advancing wave flows backwards. There are other ways of proving that, independently of friction, there must be low water under the moon; a geometrical one by the Astronomer Royal at p. 229 of vol. xxvi. of the R.A.S. 'Notices' is too long to copy here.

Now let us consider the effect of friction, which of course tends to retard the advance both of the wave and the water. If the wave's head is at E., a cardinal point, there is no lunar tangential force, and gravity acts no more forwards than backwards at the top of a wave; but the particles of water are advancing with their greatest velocity there, and therefore with the greatest friction against them. Consequently that unbalanced force of friction would carry the wave head farther back, from E. towards N. But how far will it go? Between E. and N. both friction and tangential force would urge it still backward, and gravity does nothing to help it forward; and therefore it cannot stay there. And we know it cannot be at N. for the reasons found above. But between N. and W. it can; for there the friction acts backwards or towards W., while the tangential force always acts towards N. and S., and so the forces can balance each other there and produce equilibrium.

Thus the head of the spheroid which statically belonged to S. facing the moon, has fallen back 225°

through E. and N. to N.W., and thereby the spheroid looks as if it had only fallen back 45° by friction according to the statical theory, and all the other apparent efforts upon the earth are the same. Or we may say that friction prevents the head of the wave from falling back from S. to E. but only to S.E; which is a more simple view of the matter.

As the moon does not go round the earth daily in the plane of the equator, but is above or below it except on some two days in the month, she generally pulls the tide obliquely, which makes it rather less than it would be directly. The average lunar tide or difference between high and low water is calculated under the wave theory to be barely 2 feet, and therefore the solar tide will be nearly 1; which makes the spring tide nearly 3 and the neap only 1 foot in the open sea; and this agrees with observation on solitary rocks in the ocean which impede and therefore raise the tide very little.*

Retardation of the earth by the tides.—The friction between the water and the earth must affect the earth's rotation. Suppose the earth to be still, and the moon brought into action anew and dragging the water round; it is evident that in time it would impart some motion to the earth in the same direction. And though it is not the water but the tidal wave which goes round with the moon, still the water does in some degree: as much must be always going forward as would fill up the difference between high and low tide. You may say that when you send forward waves

* As at Rockall in the Atlantic, 260 miles off Ireland.—'Gallery of Nature,' p. 199

in a rope by shaking it at one end the rope does not go forward; but some air goes forward on that side of the rope which you first move it to, and the rope only represents the surface of the wave. And as the earth rotates 27·32 times faster than the moon goes round it the friction of whatever water goes (or rather stays behind) with the moon must retard the earth's rotation.

The tide is also said by some writers on the subject to retard our rotation in another way—not that it really is another, for they both resolve themselves into friction. It is only another way of explaining the same result. The tidal protuberances being always about 45° behind our two imaginary half-moons are nearer to them than the earth's centre is, and are therefore more attracted, and so form a kind of handle or lever for the moon to hold the earth by, so far as the water is to be considered part of the solid earth, which it is just so far as they are connected by friction, and in that way the moon's attraction on the tidal protuberances must retard the earth.

This same attraction must act reciprocally on the moon, and we must consider now in what direction. What we called 'behind' when we treated the earth as non-rotating, really means eastward of the moon, and that is *before* her when we consider her real motion eastward round the earth, and not round any place on the surface of the earth, which by rotating faster makes the moon appear to go the other way. Consequently this reciprocal force tends to pull the moon forward in her orbit, i.e. to increase her centrifugal force, and therefore her distance from the earth, and therefore

her period, and therefore is practically a retarding force as regards her revolution round the earth.* This must in some unknown degree diminish the moon's 'secular acceleration,' which we shall consider presently, and shall find to be compounded of a real acceleration of the moon and a real retardation of the earth's rotation, making together an apparent acceleration of the moon twice as great as is due to the one cause (different from this) which can be estimated.

No Centrifugal Tide.—I had better notice here a sort of tidal paradox which a very great mathematician, who explained it to me, confessed had puzzled him when he was learning astronomy. It seems as if the centrifugal force round the centre of gravity of earth and moon ought to throw the water outwards on the side of the earth farthest from the moon, which we called N before and which is nearly 7000 miles beyond G (p. 125), just as centrifugal force enables you to swing round a bucket full of water if you do it fast enough. In order to measure such a tide (if it existed) I must tell you that centrifugal force = radius × (velocity of rotation)^2. Velocity (whether linear or angular, as this is,) is always reckoned per second, as all forces are; and so this velocity is 360° or 6·283 ÷ the seconds in a sidereal lunation, which are 2,360,680; and we may call the radius of rotation 7000 × 5280 feet, gravity being always reckoned in feet per second, viz., 32·2. From this you may easily find that this centrifugal force would be about a 127,000th of gravity,

* The Astronomer Royal, in the paper above referred to, professed himself quite unable to say how much it may amount to, and it must be impossible to estimate the effect of such friction directly.

which is above 30 times the greatest tidal force of sun and moon together (p. 169). So that if this force really existed, a great part of England and Europe would be submerged by this tremendous tide daily.

But in fact it does not exist; or rather, there is no more centrifugal force at N than anywhere else on the earth or at its centre E, and that is the general centrifugal force which balances the moon's attraction, just as that in our much greater orbit round the sun balances his attraction. This, in short, is only another branch of the rotation fallacy. The earth's rotation is round E, not G, and must be excluded altogether in considering this question; and so excluding it, the line EN always keeps parallel to itself while E goes round G in the month; or N and every other place in the earth describes a circle of exactly the same size as E does. Consequently there is no excess of centrifugal force anywhere, and no tendency to throw the water outwards, any more than from our motion round the sun, which gives us a centrifugal force 74 times as great as the centrifugal force of every part of the earth round an axis through the c. g. of the earth and moon, and would therefore produce a still more tremendous tide, if it produced any.

DISTURBANCES OF THE MOON.

There is scarcely one element of the orbits of the planets or their moons that is not subject to continual disturbance, by the attraction of every other body which is large enough and near enough to affect them sensibly. All these disturbances in one way or other ultimately

compensate themselves: some of them first moving the body, or its orbit, a little in one direction, and then an equal distance the other way: others producing recessions or advances of nodes or apsides, which in time work round; and there is one remarkable acceleration of the moon, which has such an enormously long period, that it may be said to increase perpetually, though the time will come for it to change. It is quite beyond the scope of an elementary book like this to describe all the inequalities (as they are called) of the moon alone, to say nothing of the planets. Here and there I must notice a few of them, as I have already precession and nutation—a disturbance upon a disturbance of the earth. The following are the most important lunar disturbances, which are explained in somewhat different ways, and at greater length, in Herschel's Astronomy, Airy's Gravitation, and Newton's Principia, where the problem was first solved. I think Airy's explanation much more perspicuous than Herschel's; but nothing of that kind can be easy. They are also explained in another slightly different way in Proctor on the Moon.

Moon's Secular Acceleration.—We saw at p. 53 that the minor axis of the earth's orbit was less, or the eccentricity greater, 20,000 years ago than it is now; and the minor axis will increase for 24,000 years yet, while the major axis remains unaltered. So the sun's average (but not *mean*) distance from the earth and moon increases, and his power to disturb the moon decreases. Now let us see what that disturbance does. The sun attracts the new moon more than the earth, and the full moon less, because of their difference

of distance: therefore at both syzygies (as the places of new and full moon are called) the sun's differential force practically diminishes the earth's attraction on the moon. When they are equidistant from the sun he draws them closer together, as you would two separated balls by pulling them with strings of equal length. But this contracting force may be proved to be only half as great as the differential force, as in the similar case of the tides (p. 169). At intermediate places both forces are evidently less; and at certain points nearer quadrature (or half-moons) than syzygy they balance each other. Therefore on the whole the differential force greatly preponderates, and weakens the earth's attraction, and so enlarges the moon's orbit; and therefore her time of performing it is longer than if there were no sun. But as his power of thus retarding the moon decreases with the increase of his average distance, she is *comparatively* accelerated the 833rd of a second a year, which accumulates to 12 sec. in a century by arithmetical progression. And she gets nearer the earth about an inch a year, or 8 feet in a century, as stated by Professor Adams in the R.A.S. 'Notices,' xx. 228.

The Annual Equation.—As this retarding force is greatest in winter, when the sun is nearest (p. 50), the moon falls most behind her mean place in April, after half a year's excess of retardation, and similarly gets 11′ 12″ before it in October. This is called the *Annual Equation;* but the mean place here referred to is the mean elliptical place, which is found by applying the *equation of the centre* (p. 51) to the mean place which she would have if she moved in a circle. Its greatest

amount is 6° 18″, which is equivalent to 12h. 24m. since the moon goes very nearly 1″ in 2 seconds.

But now comes a most remarkable result of the latest investigation of this small advance of the moon: which is *apparently* not the 12s. just now mentioned, but 24. From one cause or another she is always 24s. or 12″ past the meridian at the time when she would just be there if she had kept her mean apparent velocity of 100 years before. This was first ascertained by Halley in 1693 from a comparison of old observations, but not accounted for until Laplace explained it as I have described, nearly a century afterwards. And all the astronomers who followed him considered his calculations complete; though calculations of this kind are only approximate, and made on the principle of taking into account all the quantities which are not too small to be appreciable. But Mr. Adams, taking up the matter afresh in 1853, discovered that they had all disregarded something which was large enough to reduce the accelerating effect of the increase of the minor axis of the earth's orbit by one half. Thus half of the observed acceleration was again left unaccounted for, and where is it to come from?

Laplace's calculation, with some later corrections for other disturbances, appeared to account so well for the long observed acceleration of the moon that Adams's impeachment of it seemed an unnecessary disturbance of what was comfortably settled. Le Verrier, Hansen, and the foreign astronomers generally, except Delaunay, insisted that he was wrong. But they had eventually to admit that he was right, and that this apparent concurrence of theory and

facts was due to an accidental compensation of errors, i.e. of two overlooked causes in opposite directions.

Then came the suggestion that the required cause was to be found in the retardation of the earth's rotation by the tide (p. 181), and Delaunay calculated its probable amount; though in a case of such complicated fluid friction it is difficult to believe that any calculation of exact amounts can be relied on. The Astronomer Royal, in the paper (in R.A.S. 'Notices,' vol. xxvi.) before referred to, thought he had demonstrated that the tide can produce no such effect; but he afterwards wrote an 'addendum' to it, saying that he found it did. Indeed the conclusion is as evident without mathematics as with them, when once it has been suggested.

There still remains however the element of uncertainty described at p. 182, in the unknown amount of retardation of the moon by the same tidal action which retards the earth's rotation. All that we know for certain, by comparison of old and modern observations, is that the total apparent lunar acceleration is 24 sec. (of time) in a century, and that 12 of them are due to the diminution of the sun's average attraction on the moon by the increase of the minor axis of our orbit. The other 12 must consist of some unknown quantity, more than 12, due to the earth's rotation, diminished by some other unknown quantity due to the reciprocal tidal retardation of the moon.

If we disregard this last effect as unappreciable, the result is that any meridian, treated as a hand of the earth clock (and all our clocks only represent the earth's rotation), would be 12 seconds slow at the

end of 100 years by a clock which had gone uniformly. If you wish to know how much a day the earth must lose to produce this result, you must remember that the daily loss accumulates by arithmetical progression into this loss of 12 sec. in a century; and that makes the daily loss the 57,000,000th of a second; which (by simple addition, not arithmetical progression) makes the day a 62nd of a second longer now than it was 2500 years ago.

By the rule given at p. 112 for a long arithmetical progression you may practically say that the effect in 25 centuries is 625 times that in one century; and therefore the real and apparent advance of the moon from both causes together has accumulated to 2°, or 4 times her own diameter, or 4 hours, in 2500 years. And that might make a difference of 4 hours or 60° of longitude in the places which would see a total eclipse. The acceleration of the moon, reckoning forwards, is of course a retardation of the moon or diminution of her longitude at any given time long ago; and in like manner the earth's rotation must be quickened backwards. Suppose that without either kind of 'secular acceleration' there would have been a total solar eclipse at noon in London exactly 2500 years ago, reckoned in hours of the present length; then in fact the moon had not reached conjunction by 2 hours or 1° of her longitude, and London had passed the sun or the sun had passed the meridian 2 hours, and had passed it 4 hours by the time the moon reached the line of conjunction or of mid-eclipse; so that London probably saw no eclipse, as a total one cannot last 3 hours in this latitude all across the earth.

This sort of calculation however has to be modified, in a way that cannot be attempted except by the most skilful persons with the best lunar tables, for the variation of the place of the moon's node, which depends on the length of a lunation and on her own longitude, and it may have prevented any total eclipse anywhere just then; and the moon's distance may have been affected too, so far as to convert a total eclipse into an annular one, or *vice versâ.* And all these effects might be greater or less according as less or more of the apparent secular retardation (backwards) is due to the real retardation of the moon or to the acceleration of the earth's rotation.

Mr. Croll has pointed out another permanent effect of the tides on the moon herself.* The solar tide wave must retard the motion of the earth round the centre of gravity of the earth and moon, in the same way as the lunar tide retards the motion of the earth round its own centre of gravity, and must therefore gradually diminish the distance of the moon. For if the earth's rotation took a month, the lunar tide would only be a stationary and therefore invisible elevation of the water in one place; but the solar wave would move round it in the month in consequence of the earth's monthly revolution round the joint centre of gravity; and that must destroy some of the force of that motion, or of the earth's centrifugal force round that c. g. The moon is not directly affected thereby, but the earth is brought nearer to the c. g., and therefore their distance is diminished, and their orbit round the joint c. g. made smaller and therefore quicker. I do not know that any

* In the 'Philosophical Journal' of August 1866.

calculation has been attempted of the amount of these disturbances, and it must be much less than the others which we have been considering.

Measure of the disturbing forces.—We can easily calculate the proportion which the differential and contractive forces bear to the ordinary earth-force on the moon, at the places where they are each greatest, i.e. at syzygies and quadratures respectively. The earth's force on the moon is the mass of earth + moon, or earth × 1·0123, divided by the square of their distance, as we want to consider the earth at rest (p. 24). The sun's mean distance is about 385 times the moon's, which we will call d for shortness, and we want to see how d affects the results. Then the difference between sun's attraction on new moon and earth is the difference between $\frac{\text{sun}}{(385\,d)^2}$ and $\frac{\text{sun}}{(384\,d)^2}$, which, by a common sum in fractions, is $\frac{769\ \text{sun}}{147{,}456 \times 148{,}225 d^2}$, which again, multiplying the numerator and denominator by d, very nearly $= \frac{2\ \text{sun} \times d}{(384{\cdot}5\,d)^3}$, as you will find by trial. But the sun is 318,740 times as heavy as the earth and moon together; and substituting that figure for 'sun' you will find the differential force is to the earth's attraction, or (earth + moon) $\div\, d^2$, as 1 to 89·5. In like manner the differential force at full moon is the difference between $\frac{\text{sun}}{(386\,d)^2}$ and $\frac{\text{sun}}{(385\,d)^2} = \frac{771\ \text{sun}}{148225 \times 148996 d^2}$ $= \frac{2\ \text{sun} \times d}{(385{\cdot}5 d)^3}$ very nearly; which, with the same figure for the sun, makes the proportion of the differential

force to the earth's attraction 1 to 90·2. But when we have no need to distinguish them they are both treated as $\frac{2}{179}$, which we shall see presently is necessarily for another reason very near the mean value of the disturbing force at syzygies.

You will see the consequences of this slight excess at new moon over full presently. But you have here the proof that the differential force varies inversely as the cube of the sun's distance, and directly as twice the difference of distance of the earth and moon: all which is very like the case of the tides. Similarly we should find that the contractive force, which is half the differential when they are both at their maximum, is measured by the distance of the moon sideways from the line of syzygies, divided by the cube of the sun's distance. And the disturbing forces at all intermediate places bear some fixed proportion to the differential force, according to the position of the moon, and therefore that is the ultimate measure of them all.

It may seem odd, but it is the fact, that the magnitude of the sun's mean disturbing force on the moon depends on the proportion of the length of the year to her sidereal month of 27·32 days, which proportion is 13·37, and the square of that is very nearly 179. This is no accidental coincidence, but a necessary result of the law of gravity, which we shall see afterwards makes the square of the period of any moon or planet = $4\pi^2$ (or 39·5) × the cube of the distance of the 'primary' round which it goes, divided by the mass of the primary. Call the sun's distance D as we called the

moon's *d*. Then the differential force is to earth's force as $\frac{2\,d \times \text{sun}}{D^3}$ to $\frac{\text{earth}}{d^2}$, or (dividing both by *d*) as $\frac{2\text{ sun}}{D^3}$ to $\frac{\text{earth}}{d^3}$, or as $\frac{2}{\text{year}^2}$ to $\frac{1}{\text{month}^2}$, or as 2 to $\frac{\text{year}^2}{\text{month}^2}$, i. e. as 2 to 179. But it is an accidental coincidence that the sun's attraction on the earth is nearly 179, viz. 176·6, times the moon's.

Since the differential force, which diminishes the earth's attraction, varies from 0 at quadratures to $\frac{1}{89 \cdot 5}$ at syzygies, its mean is a 179th; and the mean of the contractive force acting the contrary way is half as much; and therefore the earth's attraction is reduced a 358th on the whole. As the moon's period^2 is inversely as the attraction or mass of the earth, any *small* decrease of that produces half as much increase of period; or the moon's period is lengthened a 716th by the sun—assuming her mean distance to remain the same. Whether it would or would not be increased, or diminished, by the imaginary abolition of the sun's attraction, depends on the relative position of the three bodies at that moment.*

Tangential and radial forces.—Now let us see what else the differential and contractive forces do, besides this lengthening of the period, subject to the small secular diminution of it. As both forces are acting at every part of the orbit except syzygy and quadrature, where they alternately vanish, they must combine to produce a resultant force in some direction between

* I have to thank Professor Adams for more information than I can use here on this point, which is either evaded or treated very imperfectly in all the books I have consulted.

them, as two winds blowing across each other would send a ship in some diagonal course between them. As the differential force always acts from the line of quadratures (QQ in the figure at p. 199), and is proportional to twice the moon's distance therefrom, it accelerates her from quadrature to syzygy and retards her from syzygy to quadrature. And as the contractive force always acts towards the line of syzygies SS, and is proportional to the moon's distance therefrom, it also accelerates her towards syzygy and retards her after syzygy. Thus a part of both these forces, whenever they *both* exist, is always resolved into a tangential force, which accelerates before syzygy and retards after it; and the rest is resolved into a radial force, which acts with the earth's attraction for 35° on each side of quadrature, and more strongly against it for 55° on each side of syzygy. At 55° from syzygy they balance each other, and the radial force vanishes; but the tangential force is greatest half way between syzygy and quadrature, at the places called *octants;* and there it amounts to $\frac{3}{4}$ of the differential force at the adjacent syzygy: but this cannot be proved here.

You will easily see that the sun is really a little farther from the moon than the earth at true quadratures or 90° from syzygy. But the difference is only $4\frac{1}{2}'$, the angle corresponding to half the moon's distance divided by the sun's; which is too small to affect any calculations that can be given here; and so is the inequality arising from the sun's force in the place of the moon's orbit being rather less according to her distance from the ecliptic, where she never is except

at the nodes. In the figure at p. 199 I have marked the forces with arrows according to their directions, and I have given the radial force at syzygies two arrows, because it is double of that at quadratures, except so far as they all vary with the moon's distance from the earth: e.g. the tangential force at A (say) 50° after S_2 exceeds the tangential force at P 50° after S_1 as much as EA exceeds EP.

Variation.—This constant acceleration up to syzygy, and retardation after it, makes the moon alternately 35′ 42″, or rather more than her own width, before and behind her mean longitude; and this is called her *variation*. You would probably expect it to carry her farther away from the earth at syzygy than quadrature. But it does just the contrary. For the moon going fastest at syzygy, from a cause different from the earth's attraction, is least drawn out of her forward course by the earth and goes farther on towards quadrature; and so the orbit becomes an oval with its sides at syzygy and its ends at quadrature, and the minor axis a 70th less than the major (supposing the undisturbed orbit to be a circle). But you must not confound this secondary oval, having the earth in its centre, with the much more elliptical general orbit of the moon, having the earth at the focus, of which this is only a disturbance.

Parallactic inequality.—We saw just now that both the differential and contractive forces, and therefore their resultant tangential force, are a little greater on the near side of the orbit than on the far side. Consequently the ‘variation’ at new moon exceeds that at full moon by 2′ 6″. This difference evidently

depends on the proportion of the moon's distance to the sun's; which may be more exactly proved as follows. If you work out the calculation at p. 191 completely, keeping the moon's distance as *d*, but increasing the sun's to 400 *d*, as it used to be reckoned, and increasing his mass to 357,050, in proportion to the cube of the distance, you will find the average disturbing force the same as before, but the difference between its two extremes a little less. And with some trouble you might find that this difference of the differential force at full and new moons = 6 × sun's mass (in each case) divided by his distance^4 (or distance × cube of distance). But since the mass bears a fixed proportion to the distance^3, that leaves this inequality to vary inversely as the sun's distance, as his parallax does. Hence it is called the parallactic inequality. It is in fact the variation of 'the variation.' And this alone of all the disturbances gives any measure of the sun's distance. Its observed excess over the amount due to the sun's old distance first led the late Professor Hansen of Gotha in 1854 to think that the distance had been over-rated; which was afterwards proved by other means to be so (see p. 85).

Subject to this small difference of 2′, the 'variation' compensates itself in opposite halves of the orbit—provided the two halves are alike in the long run; but they are not, for there is a gradual decrease in the moon's mean distance, as explained at p. 186; and this is the cause of that error in Laplace's calculation of the secular acceleration discovered by Adams in 1853. I am not aware that any non-mathematical explanation of it can be given.

The advance of the apsides may be shortly proved as follows. It is only necessary to remember first, that the sun's disturbing force on the whole weakens the attraction towards the earth.

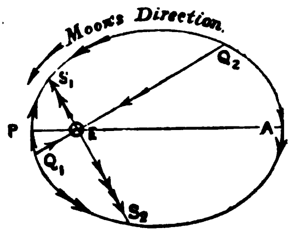
The undisturbed apogee is the place where the moon would begin to move towards the earth if there were no sun; but if the earth's attraction is weakened there it cannot pull the moon round the corner so quickly, and she will carry the apse along with her a little. At perigee she begins to leave the earth again; but if the earth's attraction is weakened there, she will begin to leave sooner than she would otherwise; or that apse comes sooner, or recedes. But these opposite effects by no means balance each other; for the differential force varies as the moon's distance from the earth, which is about a 19th greater at apogee and a 19th less at perigee than at mean distance. Besides that, the earth's attraction is itself about a 9th less at apogee and a 9th greater at perigee than at mean distance: and we found at p. 193 that the mean differential force is a 179th of the earth's mean attraction. Therefore in the long run the differential force diminishes the earth's actual attraction at apogee by $\frac{20}{19} \times \frac{10}{9} \times \frac{1}{179} = \frac{1}{153}$ but at perigee only $\frac{18}{19} \times \frac{8}{9} \times \frac{1}{179} = \frac{1}{213}$ whenever the apses are in syzygy and the effect of the forces in disturbing them is greatest. The effects of the contractive force must be opposite to those of the differential force; but as we saw at pp. 168, 192, they can be only half as great. Therefore on the whole the advance of the apses preponderates over their recession.

The sun then would drive the apses forward even if he stood still; for we have said nothing yet about his motion. But he goes round the earth the same way as the apses, and therefore drives them faster. He also stays in company with a progressing apse, keeping up its progress, longer than with a receding apse which he only meets, and thus makes them progress still more. And though the tangential force is balanced on each side of any diameter of the orbit, and therefore does not advance or retract the apses directly in the long run, yet whenever it advances them it keeps them longer in company with the sun, and *vice versâ;* and thus indirectly it augments the effects of the radial force. And by these two causes the advance of the apses is made twice as great as it would be without them, viz., 3° in a sidereal lunation. In the same way the apses of the earth's orbit are carried round by the attraction of the exterior planets; but in 110,880 years instead of 9, as their disturbing force is much weaker.

Change of eccentricity.—The sun also disturbs the eccentricity of the moon's orbit. In this figure (next page), which I have described already at p. 194, P and A are perigee and apogee for this one position of the orbit, or of syzygy and quadrature with respect to perigee and apogee: other positions would want other figures, but this will serve our purpose.

1. When the moon is at the syzygy S_1 she is approaching perigee, or the radius vector is decreasing; but the radial force there acts against it and tends to keep the radius from shortening so much, and therefore makes the orbit less eccentric (p. 49). At the opposite syzygy S_2 the radius is lengthening, and the

radial force tends to lengthen it still more, or to increase the eccentricity. We must see then which prevails. At S_2 the moon is farther off, and going slower; and so the disturbing force is both greater and has a longer time to act (p. 192): therefore the increase of eccentricity prevails thus far.



2. At Q_1 the radius is lengthening, but the contractive radial force acts against it, and therefore diminishes eccentricity. At Q_2, after apogee, the radius is shortening, and the contractive force helps to shorten it and therefore increases eccentricity. And Q_2E is greater than Q_1E, and therefore again the increase prevails. At these places there is no tangential force.

3. The tangential force, retarding from syzygy to quadrature, diminishes the moon's velocity at P, and therefore diminishes her centrifugal force, or power to fly farther off from perigee, or to increase her radius vector; and so the eccentricity is diminished by the tangential force at P. But it is increased by the tangential force retarding the moon at A and weakening her power to resist the earth's attraction, which shortens her radius faster than if she moved quicker and had more centrifugal force there. And the force is both greater and acts longer at apogee than perigee; so again the eccentricity is increased. At intermediate places near Q_2 and S_2 the effects of the tangential force balance each other pretty nearly.

The result is that all the disturbing forces increase the eccentricity when the moon has to pass through the apses before quadrature and after syzygy. You may easily infer that the eccentricity is diminished when she has to pass the apses after quadrature and before syzygy. And when they lie in either syzygy or quadrature the forces balance each other and do not disturb the eccentricity. Nevertheless it is greatest when the apses are in syzygy and least when they are in quadrature. For those places move round the earth with the sun in a year, while the apses take nearly 9 years to revolve in the same direction; therefore S_1 is approaching P whenever they are in the position of this figure, in which the eccentricity is increasing; and it goes on increasing till syzygy has reached the apse, and consequently it is greatest then, or soon after (see p. 174). Similarly it is least with the apses in quadrature. And on the whole it is half as great again with the major axis in syzygy as in quadrature.

Evection.—The variation of eccentricity and the irregular motion of the apses produce together the largest and earliest observed of all the lunar inequalities; which was called Evection, or the carrying away of the moon from her mean elliptical place by as much as 1° 20′ alternately backwards and forwards; making her oscillate through 5 times her own width in the time of the sun's passing perigee twice, or about a year and 6 weeks. Consequently it depends on, or (speaking mathematically) is a *function* of, the difference of longitude of sun and moon, and also of the true anomaly or moon's distance from perigee, on which the motion of the apses depends.

Recession of the nodes.—In all these cases of disturbance the orbit that we speak of as if it were a ring capable of being moved, is the *instantaneous ellipse*, which the moon would go on describing thereafter if the disturbances were stopped. The recession of the nodes of her orbit on the ecliptic is caused by the attraction of the sun on the moon, exactly as the precession of the equinoxes, or nodes of the equator and ecliptic, is by the attraction of the sun and moon on the equatorial protuberance of the earth, which may be considered a ring of satellites stuck together. Here we have only the motion of one satellite to consider.

But the effect of the sun and moon's attraction on the equatorial ring or 'elliptical excess' in causing precession is much less than it would be if there were not a sphere many times heavier inside, which has to be dragged round with it in giving the twisting motion to the earth's axis. The bulk of the elliptical excess is a 149th of the whole earth (an oblate spheroid and the sphere within it being in the proportion of the squares of their different axes): but the outside is not half as dense as the inside (p. 36); and therefore the mass of the whole earth is probably above 300 times that of the elliptical excess. Again, that is not really concentrated into a ring, but spread over the whole surface from 13 miles thick at the equator down to nothing at the poles. Moreover the forces which produce all the disturbances vary as the distance between the two attracted bodies, and the moon is 60 times farther from the earth's centre than the equator is. From all these causes together the nodes of the earth's equator recede 1390 times slower than those of the moon's orbit.

1. As the sun occupies all sorts of positions with respect to the nodes during a year (or rather less, since the nodes revolving backwards meet the sun again in 346·607 days) we must consider them in succession. First let us take the nodes in quadrature. Then the tangential force urges the moon forward as she rises to syzygy and to her greatest latitude from the ecliptic. Besides that, there is always a force towards the ecliptic, as the sun's differential force is always trying to pull the near side and push the far side of the moon's orbit down to the ecliptic, except when he is himself in the line of nodes and therefore in the plane of the moon's orbit. This is called the *resolved force* of the sun towards the ecliptic.

Now if the moon's apparent path in the heavens, rising from the ecliptic and coming down to it again, is opened out into a flat picture, as at p. 72, it will look like the path of a stone or a ball shot from the ground at an angle of 5°, and coming down to it again at the same angle, the place of falling corresponding to the next node. But that disturbing force of the sun which acts towards the ecliptic is the same as if a wind blew down upon the ball; and the effect of that would manifestly be to make it reach the ground sooner, or the node to recede. Also while the ball is rising, the downward force would evidently keep diminishing the angle of inclination of its course; but would increase it while falling; and so the inclination would end as it began; though the moon or the ball has not risen so high above the ecliptic or the earth, nor gone so far, as if the disturbing force had not acted.

The tangential force, which always urges the moon forward from quadrature to syzygy, tends to postpone her arrival at the next node, and also to make her course straighter or the inclination of the orbit less; but the same force acts the contrary way from syzygy to the next quadrature and node, and so those two balance each other.

2. Next let the nodes be in the line of syzygy. As the sun is then in the plane of the moon's orbit, he plainly can do nothing towards pulling or pushing the moon out of her orbit, or altering either the nodes or the inclination. And from these two cardinal positions of the nodes we may conclude, that as they recede through the whole lunation in one case, and never advance in the other, they must on the whole recede, even if they advance a little in some intermediate position.

3. But we may as well complete the inquiry by seeing what happens when they are neither at quadrature nor syzygy; and first, let each node be after quadrature. Then as the moon comes down to node from quadrature, towards the sun, he pulls her forward out of the course she is taking, and so makes the node advance, i.e. makes her reach the ecliptic later. In the opposite quarter of the orbit, the force acts similarly, and there also makes the node advance. But in the rest of the orbit the disturbing force is towards the ecliptic as before, and therefore makes the nodes recede.

4. Lastly let the nodes be before quadrature and after syzygy. Then while the moon goes up from node to quadrature, leaving the sun, he also pulls her back, which makes her course less parallel to the ecliptic, as

a head wind would make the course of a ball, while rising, still less parallel to the earth: and that postpones her arrival at the next node, or makes it advance. And the same things happen in the opposite quarter, as usual. But in the rest of the orbit the force is downwards, or towards the ecliptic, and so the nodes recede. Therefore on the whole, as the nodes cannot advance through more than two quarters of the orbit in any lunation, or through more than half that quantity on the average, and recede in all the rest, our former conclusion was right, that the recession greatly preponderates.

If we followed the inclination also through the two last cases, we should find that it is diminished when the nodes are in the third position, but increased when they are in the fourth, and therefore is not altered in a complete set of lunations, when the nodes and sun have gone all round each other, except by the minor disturbances beyond the scope of this book.

Sir J. Herschel said ('Outlines,' p. 524) that Hansen's discovery in 1847 of two lunar disturbances due to Venus 'accounted satisfactorily for her only remaining material difference between theory and observation,' i.e. after the 'almost innumerable' multitude of others previously known; and we must add, subject to the subsequent corrections of Adams and Delaunay above mentioned. One of them is caused by the variation in the distance of Venus through the eccentricities of her orbit and the earth's; and that accelerates the moon for 136 years and then retards her for another 136, reaching a maximum each way of 15″ according to Hansen's latest figures. The other is due to the near

coincidence of 13 years of Venus with 8 of the earth, which will be noticed afterwards. He calculated that that accelerates and retards the moon for periods of 120 years alternately (which period will turn up again in the transits of Venus), with a maximum of 23″ each way.

But this last conclusion has lately suffered a worse fate than Laplace's secular acceleration; for it has been reduced by recalculation, in several hands, to 'insignificance;' and though the maximum was small, the loss of it is enough to produce discrepancies from which the Astronomer Royal concludes that 'there is still some serious defect in the lunar theory.'* This is another instance of the disturbance of an apparently comfortable agreement between theory and observation by the inexorable logic of mathematics — no doubt destined to be again set right by the discovery of some new disturbance of the moon, which would otherwise have remained unknown.

* See R. A. S. 'Notices' for November 1873, and also p. 150 above.

CHAPTER IV.

THE PLANETS.

THE earth is by no means the only body that goes round the sun. In the earliest times of astronomy it was observed that there were five stars unlike all the rest in their behaviour; apparently going round the earth (independently of their daily rising and setting) in longish periods, though with some irregular motions backwards and forwards, two of them taking a year to go round on the average * and the other three taking nearly 2, 12, and 30 years. (We have now two more, taking 84 and 165 years.) Those five *wandering* stars were therefore called *planets*. The ancients either named them after some of their gods, or their gods after them: for it is by no means clear which came first, the heathen *mythology* and worship of false gods, or the belief in the planets influencing the bodies and fortunes of men; the study of which is called *astrology*. I have no doubt that came first, for reasons

* The apparent average periods of the two *interior* planets, within the earth's orbit, have no relation to their real periods round the sun, to whom they only appear as companions; and therefore their apparent returns to the same stars only vary a little from a year, backwards and forwards. The *exterior* planets, which really go round the earth and its orbit, are not so affected, and their average apparent periods correspond to their real ones. The apparent retrogradation and standing still of all the planets sometimes will be explained afterwards.

which are not material to state here. The planets have still kept these old names, and no doubt always will. For the days of inventing names are gone for ever.

The names of those five old planets are Mercury, Venus, Mars, Jupiter, and Saturn; and the days of the week are still named after them, with the addition of the Sun and Moon: either directly, as Saturday, Sunday, Monday, or through the Saxon names for the others: thus Tuesday is the day of Mars (Tuisco), Wednesday of Mercury (Woden), Thursday of Jupiter (Thor), and Friday of Venus (Friga). It is singular that the planets were not assigned to the days either in the order of their distances or their apparent size.

These five planets with the sun and moon were also the principal characters in that 'host of heaven' which the idolaters of old worshipped long before the Greeks; some of whom took Paul and Barnabas for Jupiter and Mercury, and others worshipped an image of Diana, the goddess of the moon (Acts xiv. 12, and xix). The first Greek historian Herodotus says that the first Greek poet Homer, who probably lived about the time of the prophet Elisha, borrowed the names of Jupiter and most of the other Grecian gods from the Egyptians, who also practised astrology or divination by the planets, as the Chaldæans did (Herod. II., 4, 50, 53, 82). Baal and Ashtoreth were the gods and idols of the Sun and Venus, and Chemosh probably of Saturn; and we know that the people of Israel were threatened and punished for burning incense to 'the gods of the nations,' Baal, the sun and the moon, 'the queen of heaven,' and 'the planets and all the host of heaven.' The Egyptian worship of the bull is thought by those

who have investigated such things to have arisen in the time when the sun rose for the vernal equinox in Taurus, from 4400 to 6600 years ago (see p. 76).

Indeed no one who has inquired into both subjects can doubt that pagan idolatry was connected with the belief in the influence of the sun and moon and planets, and even the stars, on the bodies and affairs of men.* There is reason to believe that the things translated *groves* in some passages of Scripture, which were 'built' and 'set up on every high hill and under every green tree,' and carried out of the house of the Lord by king Josiah and burnt, and therefore certainly not groves of trees, were wooden machines representing the planets and their apparent motions,† and used as images of the powers then supposed to rule the world, rather than the Lord who made the heavens and all the host of them, and will one day 'make new heavens and a new earth wherein dwelleth righteousness.'

When Copernicus found out that the earth goes round the sun, he found the same of the planets also; in other words, he discovered that the earth is one of the planets. And as I said before, people were at last driven to acccept his theory or explanation of the planets' motions by finding that no other would account for them; for the planets would not appear where they ought according to any of the other theories. But it was long before the reason of their motions was discovered, or even the full number of the planets. Sir Isaac Newton knew of no more than the five old ones and the earth, because the telescopes of his time were

* See Faber's 'Origin of Pagan Idolatry,' &c.

† See Landseer's 'Sabæan Researches.'

not large enough, that is, did not take in enough planet-light at a mouthful, to show the smaller planets and the more distant ones which have been discovered within the time of people now living. Copernicus did not even get so far as to discover that the planets describe ellipses, although it was known to Hipparchus 1700 years before that the sun is not always at the same distance from the earth, because his disc is larger at some times than at others. The elliptic motion was discovered by Kepler soon after the year 1600, together with two other remarkable laws of planetary motion, which I will explain hereafter. But he, like Copernicus, only found that the planets observed these laws of motion as a fact. Newton found the reason for them, and proved that every planet round the sun, and every moon or satellite round a planet, must observe them.

The dimensions, weights, and motions of the whole solar system, which means the sun with its planets and their moons, are now considered to be as follows.

1. The first planet is **Mercury** ☿, 35½ million* miles from the sun at mean distance, and going round him in nearly 88 days. By 'days' I mean our days, and not the planet's own days; for if we want to compare their periods or times of going round the sun, we must measure them all by days of the same length; and you will soon see that there is no relation whatever between a planet's period or year and the length of his day or time of rotation. Indeed it is remarkable that the four nearest and smallest planets have days of

* I give the more precise figures in the table at the end of the book where nothing turns on their precise amount. The figures have all been reduced from those in Herschel's 'Outlines' to suit the amended sun's distance, as explained at p. 85.

nearly or exactly 24 hours, and the four largest, which lie beyond a great gap in the system, have much shorter days, so far as is yet known.

His diameter must now be called 3070 miles; and so he is not quite 3 times larger than the moon, but more than 5 times as heavy, because he is rather denser* than the earth (1·144), which is 17·17 times as large as Mercury, but only 15 times as heavy. The sun is nearly 5 million times as heavy. But though Mercury is so small, he turns on his axis slower than the earth, his sidereal day being 5½ minutes longer than ours. He is always so close to the sun, never more than 29° off, that he is difficult to observe accurately, and can never be seen except as a 'morning or evening star,' just before sunrise or after sunset.

Mercury's orbit is the most inclined to the ecliptic of any (7°) and also much the most elliptical, the eccentricity being ·2056; and therefore his greatest distance is ·4666 and his least ·3075 times our mean distance. But even this great eccentricity only makes the axis minor about one 50th less than the major (see p. 48). I said that the earth moves through space 65,942 miles an hour; but Mercury goes much faster, viz., 105,720, or nearly 30 miles a second on the average, but 35 at perihelion which is 28 million miles from the sun, and 23 at aphelion which is 42 millions, calculated as I explained for the moon at p. 148.

The apparent *size* of the sun's disc to a planet, and the light and heat received there, vary inversely as the

* The density is compared with the earth's by simply dividing the mass in the 6th column of the table by the bulk in the 5th, and the specific gravity by multiplying that quotient by 5·5 the earth's sp. gr.

square of the distance; but the apparent *diameter* of the disc varies inversely as the distance only. And as the sun's apparent diameter here is 32′, you will easily calculate that at Mercury it varies from 1° 38′ to 2° 29′, and that the apparent size of the sun, and the light and heat, are from 5 to nearly 11 times as much as they are here. The apparent diameter of Mercury to us of course varies still more, viz., from 5″ when he is beyond the sun, to 12″ when he is between the sun and us.

2. **Venus** ♀, the next of the planets, has the most circular orbit of them all, its eccentricity being only ·007, and the semiaxis minor only 1600 miles less than the major; which is 66½ million miles, or ·7233 of our distance from the sun. Her period is 224·7 days, and her sidereal day 39 minutes less than ours. The orbit is inclined to the ecliptic 3° 23′. Consequently she travels about 77,500 miles an hour, and the sun's apparent diameter there is 44′, and he appears nearly twice as large as he does here. The diameter of Venus is 7824 miles, or a little less than the earth's; but she weighs one fifth less than the earth, and only a 401,840th of the sun, her density being only ·83 of the earth's. She gets twice as much light and heat as we do; and in fact her brightness prevents her from being so well observed as some of the more distant planets. It has been ascertained by experiments that Venus is ten times brighter than the brightest part of the full moon. Mercury on the contrary is a very dull reflector, being not near so bright as Venus, though twice as near the sun. They both have atmospheres, because light is refracted in passing by them. The brightness of Venus has prevented us from ascertaining how much her axis leans, or the

amount of her spheroidicity. Probably it is much the same as the earth's; but the axis is thought to lean very much more. Gravity at her surface is less in the same proportion as her mass, the distance of the surface from the centre being practically the same as here. Her apparent diameter is as little as 9½″ when she is on the other side of the sun, and as much as 61″ when she is nearest to the earth.

Venus like Mercury never appears far from the sun: 47° is the greatest angle, or apparent distance, called the *elongation*, ever made by the lines of sight from us to the sun and Venus. Consequently she never appears but as a morning or an evening star, rising a little before or setting a little after the sun; but she is sometimes visible by day, even without a telescope. When Venus and Mercury are nearer to us than the sun, they appear as crescents, like the moon when she is nearer the sun than the earth is, because we then see less than half of the illuminated hemisphere. When they are beyond the sun they appear gibbous (p. 135), because we then see more than half of the illuminated hemisphere. But the planets beyond the earth's orbit can never appear as crescents; and the gibbosity of those beyond Mars is too little to be seen, because our distance from the sun is so small compared with our distance from them. The phases of Venus were first seen by Galileo with his telescope. Thirteen years of Venus are only a day less than 8 of the earth, and in that time they have 5 conjunctions, or 5 synodical periods; which produces that disturbance of both their orbits referred to at page 204, accelerating the earth and retarding Venus a little for 120 years, and

then reversing those effects for 120 years more. This was discovered by Sir G. Airy. The transits of Venus and Mercury over the sun will be noticed afterwards.

3. The next planet is the **Earth** ⊖, of which I have said enough in Chapter I.

4. The first planet beyond the earth, or the first of what are called the *superior* planets, is **Mars** ♂. They had better have been called *exterior*, leaving the term 'superior' for the much larger but less dense ones which come after Mars and the group of small *asteroids*, and rotate more than twice as fast, notwithstanding their size, as you will see presently. His mean distance from the sun is 140 million miles, or 1·5237 of the earth's distance, and he performs his circuit in 687 days, or a little less than 2 years: therefore his velocity is about 53,300 miles an hour; and the sun's apparent diameter there is 21′; and 42 sidereal years of Mars are only 2 days less than 79 of the earth. The eccentricity of his orbit is one 11th, and it is inclined 1° 51′ to the ecliptic. He again is small, his diameter being only 4184 miles; and his density must be only ·65 of the earth's, or a little more than the moon's, as the earth is 8½ times as heavy, but only 5½ times as large; and the sun is 2,680,337 times as heavy.

Mars gives us better opportunities of seeing him than any of the planets: better than his superiors, because the nearest of them is never less than 8 times as far off as he is sometimes; and better than the two inferiors, because they are too near the sun. You will see by adding and subtracting our solar distance to and from his, that he is at one time only 48 million miles from

us, and at another 230; indeed he is sometimes as near as 34 millions on account of the great eccentricity of his orbit, when he is near perihelion and the earth as near aphelion as they can be together. At other oppositions, from the same cause, they are nearly 62 millions apart. Consequently his diameter appears 6 times larger, and his whole disc above 30 times larger at one time than the other, and his apparent diameter varies from 4″ to 21″ or more. Astronomers have been able to observe, what they have not in Venus or Mercury, the inclination of his equator to his orbit, and find it 27°½, or rather more than ours, and its inclination to our ecliptic 3° 18′. His day is nearly as much longer than ours as Venus's is shorter, viz., 24h. 37m. 23s., as lately determined by Mr. Proctor.

There are appearances of snow at his poles, which decreases in their summer; and there is something in his composition which makes him generally look red; but some parts look green, which are therefore thought to be water. There are clear indications of an atmosphere; and the spectroscope shows that it contains watery vapour like our own. His heat from the sun is less than half of ours, and gravity on his surface about the same as on Mercury, and less than half of what it is here.

At one time it was stated from some Greenwich observations that Mars is 8 times more spheroidal than the earth. I remarked in the earlier editions that this was very difficult to believe, as his velocity of rotation is rather less and his density not much less; and the oblateness of a globe once fluid varies as velocity^2 ÷ density, if the density is uniform, and the oblateness is

less if density increases inwards. It is now agreed that Mars is no more spheroidal than the earth, in fact too little to be measured on his very small size.

5. **Asteroids.**—After Mars there is a great gap among the old planets, as you see from the distance which I mentioned just now of the nearest of them, and there is no such gap in the distances beyond. But on the first day of this century, 1 January 1801, began the discovery of a batch of little planets, which have now reached 150 or more in number, after standing for a good many years at 4. They are called the asteroids, which means things like stars, but should rather have been called *planetoids*. The four first and largest are named Vesta, Ceres, Pallas and Juno. The others have exhausted the names of all the heathen goddesses, and they are now generally indicated by mere numbers enclosed in a circle, as ⑧. They are all very small, the two first being under 230 miles in diameter and most of them too small to measure. As the largest is 2200 times less than Mercury, and the moon would make 706 of it, it is evident that all the 150 together would make a very insignificant planet. They lie scattered about between the distances of 240 and 300 million miles from the sun, and their periods accordingly vary from 3¼ to 5½ years, by one of Kepler's laws, which I will explain afterwards. Some of them have orbits much more inclined to* the ecliptic than any of the regular planets, up to as much as 34°, and with eccentricities from 0 up to ·2. Gravity there must be

* In mathematical language, 'more inclined to' means what some would rather call more inclined *from*, or making a greater angle with, some other line or plane.

so small that a man could jump many times his own height on the largest of them all.

The first discoverers of the asteroids, especially Olbers, who found Pallas and Vesta after Piazzi had found Ceres, supposed that they were fragments of a planet somehow knocked to pieces. Such fragments might well take very different inclinations to the ecliptic, but must always pass through the point from which they were exploded, though it might be perihelion for some of them, and aphelion or any other place of the orbit for others, and they might have very different eccentricities, mean distances and periods.

It appears that the principal asteroids and some of the others do satisfy that condition,* but many of them do not; and the orbits are so far apart that the deviation cannot be attributed to their disturbances by neighbouring planets or each other. For this and other reasons the disruption theory has gone out of favour lately, and it is rather thought that these 'minor planets' are of the nature of very large meteors, of which it is now known that there are innumerable quantities and many systems with collective orbits within the solar system; but they are generally too small to be seen by reflection, and only become visible by incandescence from the friction of rushing through the earth's atmosphere when we come across them. I am not aware that either Olbers or any one else professed to have discovered the kind of force, either external or internal, which could knock a planet to pieces without leaving any trace of the body that 'ran it down,' or could make it explode with sufficient force

* I take this from the English Cyclopædia account of the Asteroids.

to send off the fragments in such very different orbits. The former alternative seems absolutely impossible now we know what thin things comets are.

6. After these little asteroids comes **Jupiter** ♃, a planet of a very different order from any we have seen yet, 1270 times bigger than the earth, and only about 1000 times smaller than the sun. But he, like the sun, is made of something not much heavier than water, or ·237 of the earth's density; for he is only 308 times as heavy as the earth, and a 1048th of the sun, supposing that we see his real body, or the sun's, and not a mere luminous envelope of both of them. The sun's diameter is nearly ten times Jupiter's and Jupiter's eleven times the earth's. Notwithstanding his great size he turns round on his axis in five minutes under ten hours, and consequently the centrifugal force is so great that his equatorial diameter exceeds his polar axis by a 16th. If his density were throughout what it is on the average, only a quarter of the earth's, his ellipticity would be about one 9th. Or if it increased inwards at the same rate as the earth's (whatever that may be) his ellipticity would be one 13th. As it is only a 17th, Jupiter's density must increase inwards even more than the earth's, and his outside is much lighter than water, which favours the idea that it is only a luminous envelope of some kind of cloud. That being his ellipticity, his bulk is a 17th less than a sphere of his equatorial diameter of 86,936 miles, and about an eighth more than a sphere of his polar diameter (pp. 19, 201).

Jupiter's mean distance from the sun is 478½ million miles, or 5·2028 the earth's, and his periodic time or

year nearly twelve of ours, or 4352·6 days. Consequently his rate of travelling through space is 27,180 miles an hour, and the sun's apparent diameter there is only 6′ 6″.

He stands nearly upright in his orbit; that is, his equator is only inclined 3° 4′ to it, and only 1° 19′ to the ecliptic; but the former is the inclination which affects his seasons, and it is too little to make any sensible difference between summer and winter, especially at his great distance from the sun. But as the eccentricity of his orbit is nearly a 20th, he is nearly one tenth of his mean distance, or 47 million miles, nearer the sun at perihelion than at aphelion; and as the heat varies inversely as the square of the distance, Jupiter gets one fifth more heat at perihelion than at aphelion; but on the average only a 27th as much as the earth; which would only produce a heat of 470° below zero (p. 54). As his density is nearly the same as the sun's, the force of gravity on his surface bears nearly the same proportion to that on the sun's surface as their diameters do (see p. 30), and is 2·7 times as much as on the earth; or a man on Jupiter would feel nearly three times as heavy as on the earth.

Jupiter not only is but looks considerably larger than any other planet, except Venus when she is nearest to the earth, his diameter varying from 30″ to 46″, or about one 50th of the sun's and moon's. His disc is seen in telescopes to have some dark bands or belts round it, which are always parallel to the equator, and are supposed to be clouds carried round with him; and sometimes darker spots are seen, which may be openings in the clouds. But Jupiter has a far

more important characteristic than these in his four moons or satellites, of which I will say more after we have gone through the planets themselves.

Jupiter a minor sun.—It has been observed by astronomers in different parts of the world that Jupiter is more luminous than he can possibly be by mere reflection from the sun. Some of them have gone so far as to say that he emits more light than he receives, though a good deal is always lost in reflection from the very best surfaces. Though others do not go so far as that, they agree that he emits twice and a half as much as Mars, and three times as much as the moon, in proportion to their surfaces, though the colour of a great part of Jupiter is ill adapted for reflection. Moreover the cloud belts sometimes vary in their detail, though their general aspect does not vary much. Another remarkable fact is that the satellites appear on his face as *dark* spots when they cross it, whereas if they both derived their illumination only from the sun there could be no sensible difference, or the transits of the satellites would be invisible.

Mr. Proctor* has propounded the theory, as the only way of accounting for all these phænomena, that Jupiter is still very hot, as the earth certainly was once and is yet inside: that this heat produces the cloud envelope, which makes his great apparent size and consequent lightness, as is the case with the sun himself; and then produces also storms among the clouds; and maintains such heat in them, or through them, as accounts for his ultra-reflective brightness. As the sun's heat would leave all the water of Jupiter

* In 'Other Worlds,' cap. v., and 'The Orbs around us,' p. 140.

500° below freezing, as we saw just now, it could certainly raise no clouds: they must come therefore from internal heat.

Unless Jupiter came into existence as a globe countless ages before the earth, it is certain that he would retain his heat long after the earth became cool; for if the real but invisible solid or liquid globe, which is 308 times as heavy as the earth, has the same mean density, it is still 6·7 times (instead of 11) as large in diameter, and would be much longer cooling, as a large red hot cannon ball is longer cooling than a small one, because the heat inside the globe has to make its way out through the mass outside it, of which the conducting power may be very small. In the case of our little moon, on the other hand, the unchanging state of her surface and the apparent extinction of all her volcanoes afford reason to believe that all her internal heat has transpired, or at any rate that she has become solid throughout, while the larger earth has not.

This conclusion is strikingly opposed to the older estimate of Jupiter as a mass of very cold water, as described in Whewell's 'Plurality of Worlds,' in which he pronounced such a planet only habitable by cartilaginous and glutinous floating creatures. It is odd that these large and light planets should now appear uninhabitable from heat more than cold. Though Jupiter is rather heavier than water we shall see that Saturn is much lighter. It is impossible that a globe of water could be so, for its density must be made still greater by the tremendous pressure of its own gravity within, as explained at p. 37. This is another

difficulty in the way of the old theory. But though all the large and light planets may be minor suns in the sense of emitting considerable heat and some light, there is no similarity between their clouds and atmosphere and those of the sun; since we have reason to believe that his envelopes or photosphere are anything but clouds of heated steam, though we do not know what they are.

7. **Saturn** ♄ is not much smaller than Jupiter, but is less than a third of his weight, being made of something only ·124 as dense as the earth, and only two thirds as heavy as water. His diameter is 74,100 miles, or rather more than nine times the earth's; but though he is 746 times as large as the earth, he is only 92 times as heavy, and the sun is 3502 times his weight. He is even more spheroidal than Jupiter, his polar diameter being one 11th less than his equatorial, though his density is much less and his velocity of rotation rather less; for he turns in 10½ hours. His equatorial parts are supposed to be drawn out farther by the attraction of the Ring, which I will describe presently together with his eight moons: and his oblateness is subject to an occasional variation which gives him the look of being what the astronomers call square shouldered: which all supports the theory of his envelope being cloudy. His apparent diameter is generally 18″, as he is too far off for the diameter to be much affected by the earth being in one part of her orbit or another. His mean distance from the sun is 877 million miles, or 9·539 times the earth's, and his year is 10,759 days, or 29½ of ours (the same number as the days of a revolution of the moon); consequently he moves

through space about 20,100 miles an hour, and the sun appears only 3′ 20″ wide there. It must be ninety times colder and darker there than here, and gravity on the surface very little more than on the surface of the earth. Saturn stands very differently from Jupiter, with his equator inclined 26° 50′ to his orbit and 28° 10′ to the ecliptic, to which his orbit is inclined 2° 29′; and its eccentricity is ·056. Consequently his frigid zones are larger than ours, and the poles do not see the sun for nearly 15 years alternately.

Long inequality of Jupiter and Saturn.—I have now to describe a disturbance of these two planets by each other, of which Sir G. Airy says ('Gravitation,' p. 150) that 'the calculations necessary to discover the effect of it are probably the most complicated that physical science has ever required;' so that nothing beyond a very general account of it can be expected here. If you wish to follow the subject as far as it can be carried without mathematics, you will find a longer explanation in the book aforesaid, and later ones in Sir J. Herschel's 'Astronomy,' and in Mr. Proctor's book on Saturn;* but you must not expect to find any complete explanation of such a subject easy.

The mean angular velocities of Jupiter and Saturn, being inversely as their periods, are nearly in the proportion of 5 to 2, or more exactly, 72 to 29. It follows

* It may be a public benefit to inform those who have to employ engravers that their habit of making the letters in diagrams and architectural drawings as small as they can, made some of the plates in Mr. Proctor's valuable book on Saturn useless to any common eyes, and even short-sighted eyes can hardly make them out. I am glad to see that this defect is avoided in his later books. The present fashion of pale and thin printing is equally stupid. Publishers take good care to avoid it in their own advertisements at the end of their books.

that every conjunction falls 242° 42′ beyond the last; or as if they fell on the angles of an equilateral triangle which itself revolves, the same way as the planets, at the rate of 2° 42′ in their synodical period of 19·86 years. If the angles are marked A B C in the direction of motion, the conjunctions fall in the order A C B A, coming round again to the same place at the third conjunction, except that it has then moved forward 8° 6′. The real motion is more irregular than this, because the velocity of the planets varies in different parts of their elliptic orbits. But if that were all, the variations would compensate each other, and all that would happen would be this:—as Jupiter approaches every conjunction he would be pulled forward a little by Saturn, who would be himself pulled back, and the contrary as they leave conjunction; and the effects of the disturbances would not accumulate.

But besides the unequal velocities, the eccentricity of the orbits makes the distances not quite the same before and after conjunction; and the disturbances at any series of conjunctions at different distances may or may not compensate each other. Moreover the orbits are not in the same plane, and that causes another variation of their mutual attraction 'resolved' into the plane of either of them; also the nodes where they cross are continually varying like the precession of our equinoxes. Sir G. Airy says it is impossible to do more than state that the mathematical result of all this is that for about 460 years the major axis of one planet's orbit is getting lengthened, and therefore its period (which always depends on the major axis only) is lengthened, and that of the other shortened;

and then for another 460 years the effect is reversed. Though the alteration is exceedingly small in one synodical period, yet by the end of the 460 years it accumulates, like the lunar acceleration (p. 186), into something considerable, viz., an alteration of Saturn's longitude by 48′, and of the heavier Jupiter's by 21′. The eccentricities of the two orbits are also disturbed, and the perihelion of each is made to advance and recede alternately for 425 years.

Similar coincidences of periods exist, as I said, between Venus and the earth, with 5 points of conjunction in the circle, instead of 3; and others not so close or frequent between other pairs of planets. None of them produce nearly so large a disturbance as this of Jupiter and Saturn, which accumulates for such a long time, and is therefore called either the *great* or the *long* inequality. You will see hereafter that a similar, and relatively a greater disturbance exists among three of Jupiter's satellites, whose conjunctions recur at only one place in the orbit of each pair.

It is generally said that Galileo's telescope first revealed Jupiter's moons and Saturn's ring. But it is by no means clear that they were unknown to the Chaldæan astronomers, whether they had telescopes or not, as they had single lenses at any rate (which alone will not serve for telescopes), for one has been found at Nineveh. Mr. Proctor notices in his 'Saturn' some ancient symbols of Baal or Jupiter with four wings tipped with stars, and a globe surrounded by four smaller ones; and one of Nisroch like an open eye, which is the oblique view of Saturn and his ring, besides other ringed symbols. This might not go for much if it were

certain that these objects cannot be seen without telescopes. But it is now certain that they can, where the air is so dry as to intercept much less light than it does here. In the R. A. S. 'Notices,' vol. xvi., 158, is a letter to Sir J. Herschel from a Mr. Stoddart in Persia, an observer of 15 years' experience, saying that he had several times seen with the naked eye both Jupiter's satellites and Saturn elongated into a distinct oval, as Galileo first saw it. They are also said to have been seen from Mount Etna. There is no reason against it in their apparent width or distance apart, for the planets themselves are visible as globes, and not as mere points like the stars. The only difficulty is the faintness of the light of their appendages; and that, like heat (p. 55), depends on the dryness of the air, which Mr. Stoddart said was so great that there is no dew. He also saw some stars as double, which are quite inseparable here without telescopes.

8. **Urănus.**—These are all the planets that were known until the year 1781, when a new one was discovered by the late Sir W. Herschel, who was once organist at Bath Abbey, and some other places, and afterwards the greatest astronomer of his time, and by a piece of rare good fortune the father of another not inferior to himself, and the grandfather of a third astronomer. He, like Newton and Galileo, invented and made telescopes on a plan of his own, larger and more powerful than had been ever made before; which have been since copied on a still larger scale by Lord Rosse.

With one of his smaller telescopes Herschel found a new planet farther off than Saturn, and too far to be

seen without a strong telescope, though it afterwards appeared that it had been seen before, but not perceived to be a planet. At first it was called the Georgian star, after the king in whose time and neighbourhood (viz., at Slough near Windsor) it was discovered, and by whom Herschel had been liberally assisted. But the public preferred to name it after its discoverer, until at last they both gave way to another heathen God, Uranus, the father of Saturn and grandfather of Jupiter, and whose name in Greek means the heaven itself, beyond which it was supposed there was nothing farther to be found.

This planet Uranus ♅ is 1765 million miles from the sun, or 19·1824 times the earth's distance, and takes 84 years and 6½ days, or 30,987 days, to go round him, at the rate of 14,640 miles an hour. He also ranks as a large and light planet; for his diameter is nearly half Saturn's, or 34,070 miles, and he is 71 times as large as the earth, but not quite 16 times as heavy, his density being ·22, or rather less than Jupiter's. He is 20,470 times lighter than the sun. His orbit is the nearest to the ecliptic of them all, being only inclined 46′ 29″; and its eccentricity ·046. He has four, and some think seven or eight moons, which behave differently from all others in the solar system, having their orbits so nearly perpendicular to the ecliptic as 79°, and nearly circular, and moving the opposite way to all the other moons and planets. His apparent diameter is only 4″; and consequently his inclination and time of rotation are not yet ascertained. But his spheroidicity is thought to be as great as Saturn's; and it is most likely that his equator is nearly perpendicular to the ecliptic, as

the orbits of Jupiter and Saturn's moons nearly coincide with the equator of their planet. This also accounts for his appearing sometimes spherical and sometimes spheroidal, according as his pole or equator is presented to us. The light and heat there must be 330 times less than here; gravity one seventh less; and the apparent diameter of the sun 1′ 42″ to the Uranians.

9. **Neptune.**—Still the solar system was not exhausted, as it was supposed to be when Uranus was so named. All these planets were discovered by being seen, and seen to move; but one more was proved to exist without being seen, and afterwards found by looking for it where the discoverers said it would be found. The history of this discovery is so remarkable, and at the same time is given so imperfectly, and sometimes so unjustly, in larger books than this, that I shall relate it more fully than would otherwise be necessary, taking my account of it from the most authentic source, viz., the correspondence and statement of the Astronomer Royal, in vol. xvi. of the Astronomical Society's 'Memoirs,' or vol. vii. of their 'Notices'; which very few writers of the English language, on either side of the Atlantic, and of course no French ones, have apparently taken the trouble to read before publishing their own versions of the transaction. Fortunately there never was any dispute about the facts. But the only fair accounts that I have seen are in Grant's 'History of Physical Astronomy,' and in a book called *Speculum Hartwellianum* by the late Admiral Smyth, P.R.A.S.

Just forty years after the discovery of Uranus astronomers began to complain that he did not appear in his proper place, as calculated from the earlier observations,

by which his orbit was supposed to be as well ascertained as that of any other planet. And some people went so far as to doubt whether Newton's law of attraction might not be subject to variation at so great a distance. By the year 1830 his longitude, or distance from the equinoctial point ♈, had got wrong by 30″, which would make him appear wrong in his time of crossing the meridian of any observatory by two seconds of time; and he had got 2′ or 8 seconds wrong in 1845. This does not seem much to be disturbed about in a planet 2760 million miles off, or to make the laws of the universe suspected by some astronomers, and the existence of an unseen disturber of the peace of Uranus by others. But so it was; and it gives you some idea of the accuracy now expected in astronomy.

The first person who appears to have openly suggested the idea of Uranus being disturbed by a more distant planet was the Rev. T. J. Hussey, of Hayes, who wrote to the Astronomer Royal to that effect in November 1834; and he said that two foreign astronomers, A. Bouvard and Hansen, agreed with him. But Sir G. Airy answered that he 'did not think the irregularity of Uranus was in such a state as to give the smallest hope of making out the nature of any external action on the planet,'—if there was any, which he doubted; and preferred supposing that the earlier observations had been wrong. In 1837 E. Bouvard, the nephew of A. Bouvard, again wrote to Sir G. Airy suggesting the same cause; who again answered (in substance) that he did not believe it, and added—'if it be the effect of any unseen body, it will be nearly impossible ever to find out its place.' By 1842 Bessel and other eminent astro-

nomers seem to have avowed the same opinion as Dr. Hussey, but without convincing the English Astronomer Royal, who still appears to have had no solution of his own, except his guess at the inaccuracy of observations, although the attention of astronomers had now been directed to it for twenty years, and the error had been getting worse.

But suggesting that there must be a planet somewhere was a very different thing from setting to work to calculate whereabouts in all space it must be, with a strong presumption only, from the distances of the others, that it would be nearly twice as far from the sun as Uranus (which after all it is not), and near the ecliptic like the rest (which it is). It was also certain that the disturber of Uranus had left conjunction with him, because the disturbances had begun to diminish again. In 1844 Professor Challis, the head of the Cambridge Observatory, wrote to ask the Astronomer Royal for some Greenwich tables of Uranus for a 'young friend of his, Mr. J. C. Adams, of St. John's College, who was at work upon the theory of Uranus.' He of course sent them, and in September 1845 Professor Challis wrote again to say that 'Mr. Adams had completed his calculations of the perturbations of Uranus by a supposed ulterior planet.' In October 1845 Mr. Adams himself left at the Greenwich Observatory what Sir G. Airy justly called afterwards 'the important paper,' giving the result of his calculations and *the place where the new planet would probably be found.*

Still Sir G. Airy could not believe that a young man, who had only taken his degree (of senior wrangler) the year before, had actually found the place of a planet

which he believed not to exist at all, and to be 'nearly impossible to find' if it did. So instead of encouraging Mr. Adams, or taking steps to get the planet looked for by the best telescopes of various observatories, he sent him a question which he called an *experimentum crucis*, or what is popularly called a posing question. He afterwards expressed his 'deep regret' that Mr. Adams did not answer it, which it is well known that he could easily have done, as his previous calculations involved the answer. The world has never been informed what delayed it: whether Mr. Adams wished to send some further information with it, or whether the maker of what has been called the greatest astronomical discovery since Newton felt that his announcement of it might have been received more cordially.

But while Sir G. Airy was waiting to believe in the discovery an eminent foreign astronomer stepped into the field and confirmed it; for in June 1846 M. Le Verrier gave to the French Academy his own independent calculations for a new planet, which nearly agreed in their result with those which Mr. Adams had given in 1845. As soon as they came here Sir G. Airy's doubts vanished; and then he confessed (what indeed his question showed before) that he had doubted the accuracy of Mr. Adams's investigations until he received M. Le Verrier's confirmation of them; which does not mean that he considered the calculations wrong in any definite way, but simply that he doubted any man's ability to make them.

Then he did set to work to get the planet looked for in the place indicated; and Professor Challis undertook the search with the great Cambridge telescope (which

he ought voluntarily to have done a year before) and soon found what turned out to be the planet, and he noted it as 'appearing to have a disc,' which only planets have. But unfortunately he had no star map to compare his observations with; and also delayed comparing his own successive observations with each other, until Dr. Galle of Berlin had not only found the planet, but found that he had found it, on 23 September 1846. Not that the finding of a planet where you are told to look for it is any great feat, or at all parallel to Herschel's finding of an unsuspected planet.

Only one more sentence need be quoted from Sir G. Airy's certainly candid statement to complete the story, and to show how this country lost the *undivided* credit of this great discovery, which Adams unquestionably first made and first disclosed, not merely to private friends, but to the two official heads of astronomical science in England. Sir G. Airy concluded his account by saying, 'I consider it quite within probability (he might have said, certainty), that a publication of the elements (of the planet's orbit) obtained in October 1845 (from Mr. Adams) might have led to the (telescopic) discovery of the planet in November 1845,' seven months before Le Verrier disclosed his calculations.

The name of Neptune ♆ was soon given to the new planet, on the same principle as the others; only they were obliged to go back to the brother of Jupiter, as he had no more ancestors in the Pagan mythology. Neptune was afterwards found to be 2763 million miles from the sun, or 30·0363 times the earth's distance, and to have a period of 60,127 days, or 164⅔ years. He

only goes 10,560 miles an hour, or just ten times slower than Mercury; and the Sun appears there only 1′ in diameter, or no larger than Venus sometimes does to us. Neptune's diameter is believed to be a little larger than Uranus's, viz., 38,400 miles, and his bulk consequently 117·5 times the earth's. The present estimate is that his weight is nearly 18 times the earth's, or that the sun is 18,780 times his weight. His orbit is almost as circular as Venus's, and 1° 46′ inclined to the ecliptic. Nothing is known of his time of rotation, or the inclination of his axis; but though his apparent diameter is only 2″, he is said to be visibly spheroidal, which implies a quick rotation. The light and heat there are only a thousandth of what they are here, and gravity about a quarter less than on the earth's surface.

One satellite of Neptune has been already discovered by Mr. Lassel, revolving in 5d. 21h., and going more distinctly retrograde, or opposite to the usual direction, than the satellites of Uranus, because its orbit is only 29° inclined to the orbit of the planet. It will be curious to ascertain whether his rotation is retrograde also. It also turns out that the planet had been occasionally seen before as a very faint star as long ago as 1795; but as it was never seen twice in the same place, the observations had been hastily treated as mistakes; a warning to all men never to disregard any new fact, until they are quite sure that it is not one, or is really unimportant; besides that other warning to men in high places, which the history of the discovery clearly enough proclaims.

10. One more planet was for a time suspected to exist, a very small one, only 14 million miles from the sun, and

going round him in $19\frac{3}{4}$ days, in an orbit $13°$ inclined to the ecliptic, and the name of Vulcan was assigned to it. But its existence is no longer believed, as it has been seen no more.

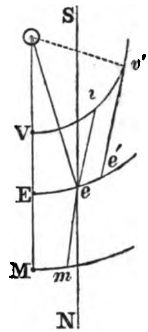
Bode's law of distances.—I have several times spoken of the distances at which the new planets of this century were expected to be found, from the proportionate distances of the older ones; and you will see in running over them that there is a rough approximation to a successive doubling of the distances of all beyond the first. If you divide them all by 9 millions, to get rid of a great number of figures, they will come in pretty nearly the following proportions: Mercury 4, Venus 7, Earth 10, Mars 16, Ceres and Pallas and some of the other asteroids 28, Jupiter 52, Saturn 100, Uranus 196, Neptune 305. Or Earth is twice as far beyond Mercury as Venus is, Mars 4 times as far, the Asteroids 8, Jupiter 16, Saturn 32, Uranus 64; but Neptune is not 128 times as far, but only 102; that is, he is 700 million miles too near according to this rule, which is called Bode's law, from its discoverer. And so the only thing like a rule, which there was to guess Neptune's distance by, turned out wrong. Nevertheless it had been useful as a first approximation; for it is a common practice in astronomy, when several things are unknown together, to assume a probable value for one of them, and then correct them backwards after some of them are found. Moreover the direction in which Neptune was to be looked for was rightly calculated, and the error in distance affected his apparent place very little, as he was then nearly in a line with the sun and earth, and his distance is 30 times the earth's.

You must understand that this rule of Bode's is only *empirical*, which means founded upon trial or experience —more or less extensive as the case may be, and is not deduced from that great law of universal attraction or gravitation, which keeps the planets in their orbits moving with a certain velocity, and makes their years bear a certain proportion to their distances, and enables their weights to be calculated from their disturbances of each other and of their moons, and an unknown planet to be found by calculation, and raises the tides, and prevents the whole earth and the sea, and everything that is in them, from flying to pieces like the wringings of a mop, and running off in straight lines into infinite space for ever. Nevertheless it is all but impossible that so many coincidences with that law should be accidental, and not the result of some yet unknown physical cause; and we shall see more of it among the satellites of the greater planets.

Retrogradation of planets.—Although the planets seen from the sun would appear to go round him, as they do, with very little variation in their pace, they appear from the earth to go very differently, and irregularly in speed, and sometimes actually to go backwards. The angular velocities round the sun depend simply on the periods: so the angular velocity of the earth is always nearly 165 times Neptune's, and 84 times Uranus's, and 29½ times Saturn's, and 12 times Jupiter's, and twice Mars's, and two thirds of Venus's, and a quarter of Mercury's. But the angular velocities round the earth follow no such rule, and indeed no rule at all that can be expressed without a great deal of calculation: all we can do is to explain

why both the interior and exterior planets sometimes appear to us retrograde in their motion.

Suppose we are sailing westward down the middle of a wide river with a slight bend to the south in a ship called Earth at E, opposite the Sun inn ⊙ on the south bank; and that at V, between us and the sun, there is a ship called Venus going the same way, but faster; and beyond us at M another ship called Mars going the same way but slower: so that in a little time we shall have got respectively to *v e m*; N and S being north and south by the compass which we carry with us. Then, as we are unconscious of our own motion, the sun will appear to have gone a little round us to the left, through the angle S*e*⊙, and Venus to the right through the angle S*ev*; and if we turn and look at Mars, that also will appear to have gone round us to our right through the angle N*em*. You must remember that the line N S which is carried with us is our only datum line to measure motion from, and those letters indicate some stars at an infinite distance, or they may be ♈ and ♎, from which longitude is reckoned. Therefore an interior planet apparently goes the opposite way to the sun, or retrograde, whenever it is between us and the sun (which is called *inferior* conjunction), and an exterior planet whenever it is in opposition to the sun. After that, we and Venus move on to *e′* and *v′*; and since *vv′* is greater than *ee′* (because the inner planet goes faster both in angular and linear velocity, but the inner orbit is most curved) the line *ve* for a few days



moves practically parallel to itself, or *v′e′* is parallel to *ve*; and then neither planet can perceive any motion in the other with reference to the stars or ♈; and so all planets, interior or exterior, appear stationary for a few days before and after each retrogradation. At other times they appear to go round the earth the same way as the sun, though with varying velocities according to their position.

SATURN'S RING AND SATELLITES.

Saturn's Ring, treating it first as a single bright ring, is very thin and flat, like a ring stamped out of a card, having an outside diameter rather more than twice as wide as the planet, or 167,600 miles. Its inside diameter is 109,536, or only 17,713 miles from Saturn's equator; and therefore the breadth of the ring itself is about 29,000 miles. It is so thin that it is difficult to say what its thickness is; but it is considered to be not more than 100 miles. And the strangest thing about it perhaps is that it seems continually to get thinner and wider, and sometimes breaks out into fresh divisions apparently, besides the long recognized and well marked divisions into two: of which the outer ring is 9960 miles wide, the inner 17,680, and the space between them 1390. It is useless to profess great precision in these figures, from the nature of the rings and their distance.

If Saturn stood like Jupiter, with his equator (and ring) nearly in the plane of the ecliptic, we should have known next to nothing about the ring; for we should have seen nothing but two bright lines like handles, each of them about as long as Saturn's radius. Sometimes the satellites appear like beads threaded on the

thin line of the ring. Sir W. Herschel found, by observing the motion of certain lumps or inequalities on the edge of the ring, that that part of it at any rate revolves round Saturn in 10h. 32m. 15s. or 3 minutes longer than his own time of rotation. But as Saturn's equator and ring are now* inclined 28° 10′ to the ecliptic, we sometimes see it in oblique perspective, and therefore as an ellipse, getting a very good sight of the whole width of the rings and the spaces between them each and the body of Saturn. If you want to realize this, get somebody to hold a globe in the ordinary wooden frame at some distance from you; when your eye is in the plane of the wooden horizon you can only see its edge; but when it is inclined a good deal, you can see the whole of it except the part behind the globe, and you see also the space between them. The minor axis of the perspective ellipse of the ring when most elevated is ·47 of the major axis.

But the sun also must be elevated above the plane of the ring, and on the same side as the earth, though not necessarily as much, to illuminate the side of the ring facing the earth, to enable us to see it. You will find in Mr. Proctor's book pictures of Saturn and the rings in all possible phases, with a variety of interesting descriptions of them which would be out of place here. He shows too that the usual statement, originally

* The inclinations of all the planets' equators to the ecliptic depend (1) on their inclinations to their own orbits, (2) on the inclination of those to the ecliptic, and (3) on the position of their nodes or equinoctial points; but all the nodes revolve so slowly, and the inclinations of the orbits are so small, that I do not complicate the descriptions with any further notice of these changes; but I have given the present longitudes of perihelion in the table at the end.

Herschel's, that each hemisphere suffers an eclipse of 15 years by the ring, is erroneous, in the sense of a continuous eclipse; though each place may pass behind the ring, as seen from the sun, daily. Some places however do suffer a continuous eclipse of nearly 7 years in each of Saturn's years ($=29\frac{1}{2}$ of ours), and other shorter ones, besides the total darkness within his large arctic circles for various periods up to nearly 15 years (p. 221). These large planets must have very odd kinds of inhabitants, if any, considering their probable condition (p. 219) and these long eclipses.

The mass of the ring has been calculated as the 118th part of that of Saturn, from its effect in disturbing some of the satellites. And that agrees pretty well with the estimate of 100 miles of thickness, on the assumption that its average density is the same as Saturn's own. There have been various speculations as to its composition. Laplace proved that it could not be in one solid flat piece, but must be at least divided into two; and the same kind of reasoning has made it necessary to carry the division still farther. For the outer and inner parts of a ring of anything like that width require different velocities and periods to preserve their equilibrium, according to well known laws of motion which will be explained in the next chapter. If the inner part only went as fast as the outer, its centrifugal force would not be enough to keep it from being dragged into Saturn by attraction. For the ring itself is far too thin and weak to be able to hold itself together by its own cohesion or internal attraction, against such a force as that tending to pull it in pieces. Moreover the outer edge revolves in the proper time for

a satellite at that distance; but too slow for one at the inside, or even at the middle of the breadth of the ring.

But a far more serious objection to the rings being rigid at all was made by Mr. J. C. Maxwell of Trin. Coll. Cambridge, in the Adams Prize Essay of 1857; viz., that, in order to preserve its equilibrium in revolution, each ring must be so uneven in density as to have its centre of gravity more than 9 times farther from the light side than the heavy one, if it is rigid: which is completely at variance with observation —unless there is that enormous and improbable latent difference in the opposite sides. Moreover the rings would then be much nearer the planet on one side than the other, which they are not, though they are a little. You may ask how a ring can be stronger for being *limp* than rigid, provided that it is of the proper shape, whatever that may be. The answer is that there is no proper rigid shape, except that defined by Mr. Maxwell; which certainly does not exist; and if it did, it could not maintain itself against disturbances. Consequently a set of rigid narrow rings is no less impossible than a single wide one.

The idea has been therefore entertained that the rings might be fluid, so that all their parts could move along each other as they pleased. But to this also it is objected that the constant changes of motion would cause waves, which would break the rings in pieces, for the effect of waves lasts long after the force has passed away which raised them. Moreover the appearance of the rings contradicts that theory. For besides the one, or we must now say two, evident divisions, there are indications of more, and also of an inner ring,

darker than the others, but semitransparent, as if composed of bodies near together but not too close to see through. And in fact that is now taken by astronomers to be the constitution of all the rings of Saturn. We may consider them consisting of a vast number of rings, each of only one satellite in width, independently of the wider separations into three distinct rings.

Another fact quite inconsistent with rigidity, is that the rings as a whole have been getting thinner, and about one sixth wider, the inner parts coming nearer to the planet, but the outer not altering, in the 200 years since Huyghens first measured it; and more rapidly in the 80 years since Sir W. Herschel's time than in the 120 years before. Galileo saw the ring but could not make it out, though he did distinguish Jupiter's satellites with his second telescope in 1610.

Saturn's moons are far less interesting than his rings. It is enough to say here that the eighth and last in the order of discovery, but the seventh in distance and a very small one, was only found in 1848, and oddly enough, by Mr. Lassell in England and Mr. Bond in America on the same night. The sixth is much the largest, and so it has been called Titan in Sir J. Herschel's renaming of them all by names instead of numbers—Mimas, Enceladus, Tethys, Dione, Rhea, Titan, Hyperion, Iapetus. But though some of these were the 'giants' of the ancient world who fought with Jupiter, they are very small satellites of Saturn now, some too small to be seen except with the best telescopes; and all of them together capable of reflecting very little light upon the planet, only about a 16th of what we get from our moon. And Jupiter's

four give him about the same. Iàpetus is one of the larger ones, and much farther off than the rest, being 32 of Saturn's diameters distant from his centre. Our moon's distance is 30 times the earth's diameter; but then Saturn's diameter is nine times ours. Their distances and periods, which is all that seems accurately known of them, are expressed shortly in the following table. The first five follow Bode's law pretty well but not the others.

SATURN'S MOONS.	DISTANCE FROM SATURN.		Period.	Discovered.
	In miles.	In radii of ♄.	d. h. m.	
1 Mimas . .	124,500	3·36	0 22 37	1789
2 Enceladus .	159,700	4·31	1 8 53	
3 Tethys . .	197,855	5·34	1 21 18	1684
4 Dione . .	253,442	6·84	2 17 41	
5 Rhea. . .	353,647	9·55	4 12 25	1672
6 Titan . .	820,543	22·145	15 22 41	1655
7 Hyperion .	992,280	26·78	21 7 8	1848
8 Iapetus . .	2,384,153	64·36	29 7 54	1671

The distances of the only four satellites of Uranus which can be considered certainly to exist, are about 123, 172, 283, 378 thousand miles; of which the differences are 49, 111, 195, making a rough approximation to Bode's law.

Jupiter's four satellites are of far more importance than Uranus's, or Saturn's eight. They are all of a substantial size: the first, second, and fourth about as large as our moon, but a good deal lighter, having respectively one third, two thirds, and half the moon's density; and the third having a diameter and weight about half as much again as the moon, and three

times her bulk. That satellite also has nearly two thirds of the density of the moon, and half as much again as Jupiter's own. These variations of density among the planets and their satellites are remarkable, following no apparent law whatever.

Their distances from the centre of Jupiter are respectively about 6, $9\frac{1}{2}$, $15\frac{3}{4}$, and 27 times his radius; which follow Bode's law very well, the differences being successively about 3, 6, 12. Consequently the first appears to people on his surface (if there are any) about as large as our moon does to us, the second and third about half as wide, and the fourth a quarter as wide, or one 16th as large. But the largest of Jupiter's moons has only the 11,300th of his mass, while our moon is nearly one 80th of the earth. Their elements are more fully given in the table at the end of the book.

They all move so nearly in the plane of Jupiter's orbit (which also nearly coincides both with his equator and our ecliptic) that they are eclipsed every time they pass, except that the fourth escapes sometimes. And they, like our moon, always show the same face to their planet. The largest of them looks rather less than Neptune, and the apparent diameter of the smallest, at our mean distance from them, is under 1″.

The period of the first is $42\frac{1}{2}$ hours, of the second $85\frac{1}{4}$, of the third $172\frac{3}{4}$, and of the fourth 16d. $16\frac{1}{2}$h.; so that the periods of the first three are successively a very little more than double of each other. And from that coincidence of periods several remarkable consequences follow. Whenever the second and third are in conjunction, the first is exactly opposite to them; and the

place for conjunction of the first and second will be opposite to that for the second and third. Consequently they cannot all be either eclipsed from the sun, or hidden from us by Jupiter, or made invisible by being in front of his disc with the sun shining on both it and them. But two may be eclipsed or hidden while the other is so made invisible; and as the fourth may be anywhere, that may also be invisible at the same time in any of those three ways; but that happens very seldom.

Another consequence is, that all the three orbits are mutually affected by the disturbances, or strongest mutual attractions, constantly recurring at the same place for the first and second, and at the opposite place for the second and third (which aggravates the effect upon the second). The effect is to make the first and second orbits ellipses with the apses always on the line of conjunctions; and a kind of secondary major axis or pair of apses of the third orbit also: only that happens to have a larger independent eccentricity of its own in another direction. But the *perijove* of the first and the *apojove* of the second are at their place of conjunction; and the second's perijove and the third's apojove are at *their* conjunction; or the three orbits are pushed away from each other at the two places of conjunction.

But the periods are not *exactly* double of each other; and that makes the line of conjunctions and of apses revolve slowly backwards, as each conjunction comes a little before the place of the last; and it takes 486½ days for them to work round, which make 68 periods of the third satellite, and 137 of the second, and 275 of the first. This is something like the 'great inequality' of Jupiter and Saturn (p. 222): indeed this is relatively

greater, for it takes many more periods of the disturbed bodies to work round. Moreover here *every* conjunction falls near the same place, but only every *third* conjunction of Jupiter and Saturn, and the two intermediate ones rather counteract than aggravate the effect. This subject is worked out at greater length in Airy's 'Gravitation' than we can afford it here; and it is not an easy one.

The satellites of Jupiter, and Saturn too, suffer another disturbance from the great oblateness of those planets, and from Saturn's ring. A sphere would be turned into an oblate spheroid by shaving pieces off the poles and laying them round the equator. Two pieces taken from the pole and laid on the near and far sides of the equator will attract a distant body in the plane of the equator more than at the poles, because they are brought into the direct line of attraction; and the one on the near side gains more by the shortening of its distance than the other loses by the lengthening it, as at p. 167. The other pieces removed from the poles to the equidistant sides of the equator neither gain nor lose in attraction. Therefore on the whole a sphere gains in distant attraction in the plane of its equator by being turned into an oblate spheroid, though the attraction *at* the equator is less because the distance from the centre is increased (p. 46). *A fortiori*, if the spheroid becomes a flat plane, its attraction in that plane must exceed that of a sphere of equal mass: and if the middle, or most spherical part, of the plane is cut out, leaving a ring, the ring's attraction must still more exceed that of a sphere of equal mass.

This excess must evidently bear some inverse relation

to the distance. In the case of a spheroid at a considerable distance it is demonstrable that it practically varies inversely as the square of the distance; and that the distance of the first moon being 6 times Jupiter's radius, his attraction on it would be thus increased by nearly a 24th of his ellipticity, or a 408th, and would shorten its period an 816th, if his outside were as dense as his inside: which it is not, and therefore the effect is less. The effect is less on the fourth moon, which is 27 radii off, in the proportion of 6^2 to 27^2, or not one 20th as much. Therefore also each moon is less accelerated at apojove than perijove; and that *comparative* loss of attraction at apojove makes their apses advance as the greater loss of earth's attraction at apogee than perigee makes the apses of our moon advance (see p. 197).

The spheroidicity of the earth produces similar effects on the moon, but too small to be appreciable in her period, because the earth is much less oblate than Jupiter, and our moon's distance is 60 times the earth's radius. But on the other hand, she goes much higher above the equator than his moons do, viz. $28\frac{1}{2}°$; and consequently she suffers another disturbance in return for her disturbing our polar axis by nutation (p. 74). For the polar attraction of an oblate spheroid at a given distance falls short of that of an equal sphere twice as much as the equatorial attraction exceeds it, though the attraction *at* the pole is greater (p. 46). Therefore the earth's attraction on the moon is greatest when she is on or near the equator; and thus she is disturbed both in latitude and longitude by the earth's ellipticity: which can be calculated from the amount of

these disturbances, and is found to agree with the result obtained by other means.

Besides the eclipses of Jupiter's satellites, and their *occultations* behind him, they may *transit* over his face, either as dark or bright spots, according to the position of the sun, and according as they pass over a dark belt of Jupiter or a bright part of his face; or they may cast their shadows as dark spots upon him while their bodies appear as bright ones, either on his disc or beyond it. I cannot go further into these phænomena, but will tell you of two discoveries made through these moons.

Velocity of light discovered.—Some years after Galileo had discovered the satellites, which however are believed to have been known in very early times (see p. 224), their eclipses were observed always to come too soon when Jupiter was at his nearest to the earth, and too late when he was farthest off, which was certainly about half as far again, whatever the actual distances in miles might be, which were not then at all accurately known. The extreme difference between the early and the late eclipses was no less than 16m. 26s., a variation far too great to be tolerated or attributed to mistakes even in the early days of astronomy. Accordingly Römer, a Danish astronomer, in 1675, hit upon the solution that the light itself takes some definite time to come, and spends those 16½ minutes in coming across the whole width of the earth's orbit, or the difference between the nearest and the farthest distances of Jupiter and his moons from us. Therefore the light takes 8¼ minutes to come here from the sun, which is at half that difference of distances. We will now see how the longitude of places on the earth

may be found by means of Jupiter's satellites, besides another method depending on the moon.

Finding the longitude.—The longitude of a place is simply the difference between the local clock time and the clock time of Greenwich, turned into degrees at the rate 4 min. to 1° (p. 11). The time of any place is that of a clock which is at 12 when the sun is on the meridian of the place, subject to the correction called the *equation of time*, which is the same for all places, and is given in the almanacs for every day (see p. 78). The time of noon at any place may be found from the fact that the sun is equally high above the horizon, and shadows are of equal length, at equal times before and after noon; besides other methods involving more mathematics, and by the stars as well as the sun. The beginning or end of an eclipse of a satellite of Jupiter, which happens very often, may be used as a common signal to be observed at Greenwich and the place in question, as much as if it were a rocket visible to them both, or the local time of day telegraphed to the other. Instead of the eclipses being observed at Greenwich, they are calculated for Greenwich time, and published several years beforehand in the Nautical Almanac; and so when you have observed the local time of an eclipse anywhere, you can at once see its difference from the Greenwich time. But after all this method is little used in navigation, because these eclipses cannot be observed accurately enough at sea.

Longitude by Lunar Distances.—The apparent distance of the moon from the sun, or any convenient star, can be measured by the seaman's instrument, called a *sextant*, first invented by Newton, in which you

see one object coinciding with a second reflection of the other from two small mirrors, when one of them is turned through the proper angle, which you can then read off; and a man can hold it in his hands steadily enough.* This apparent distance has next to be turned into *true distance*, which means the apparent distance as it would be seen from the earth's centre, by calculations for which tables are provided; and the local time of the observation must be taken. The true distance of the moon from the sun and some suitable stars is given in the Nautical Almanac for every 3 hours of Greenwich time through the year; from which the Greenwich time corresponding to the observed distance (reduced to true) can easily be found; and its difference from the local time of the observation is the longitude of the place.

If you have some chronometers keeping Greenwich time, no other observation is wanted but that of the local time of noon, or of any star transit whose Greenwich time is in the almanac. It is not safe to rely on one watch, and if two differ you do not know which is most right; but three give a tolerably safe average, if they are good ones; and of course the more you have the better. Sometimes a great many are used for fixing the longitude of important places, and are carried backwards and forwards between them and Greenwich. The parliamentary reward of 20,000*l.* was given to John Harrison in 1767, for making the first chronometers that would find the longitude within 30 miles after a long voyage; and they have been very much improved since then.

* See Newton's paper in Ph. Tr. vol. 43, and *Sextant* in English Cyc.

Finding the latitude is a more straightforward operation, and there are several ways of doing it. For the latitude of a place is the altitude of the pole above the horizon, or the mean of the greatest and least altitudes of any star, or the angular distance of the equator from the zenith, along the meridian; which is equal to the sun's zenith distance at noon, added to his *declination* or distance from the equator, or 90° minus his *polar distance*, which is in the almanac for every day.

MEASURING OF DISTANCES BY PARALLAX.

All the modes of measuring the distances of the sun, moon and planets (except by the velocity of light, which is capable of no great accuracy) depend on one principle, viz., that of ascertaining their parallax, or apparent change of position with reference to some other body when looked at from places far apart on the earth. Our two eyes unconsciously measure distances of things not too far off by parallax; i. e. when the distance between our eyes ÷ the distance of the object is not too small an angle to be sensible. If you see anything straight behind a window bar with both eyes, you will see it on the right or left according as you shut the left or right eye, i.e. it is moved by parallax. That is the general meaning of the word, but it is conventionally understood that for objects in the solar system the two places of observation are to be at the distance of the earth's radius apart, or reduced thereto, so that 'parallax' when used alone means the angle represented by the earth's radius ÷ the distance of the body. For example, the moon observed from two

places 7000 miles apart (reckoned through the earth) makes different angles with any star properly situated for measuring, i. e. in the same plane as the moon and the two places. Those angles differ by 108′ or $\frac{1}{34}$; and therefore the moon's distance is 34×7000 miles; but what is conventionally called the moon's parallax is the angle not of this accidental 7000 miles $\div$ moon's distance, but of 4000 miles so divided, which is 57′. It is best to use the longest distances apart we can, because that gives less risk of error.

The distance of Venus, the nearest planet, might be got in the same way, but for the fact that she is never high enough above the horizon, without the sun, to escape being so much affected by the refraction of our atmosphere that such observations could not be relied on, and the parallax of Venus is always a very small angle compared with the moon's, and therefore a slight error affects it far more seriously. But Mars is free from that former difficulty, and there are times, as we have seen, when he is within 40 million miles of the earth, or not much farther off than Venus; and though that is 166 times as far as the moon, and therefore his parallax is 166 times less, still it is measurable with proper care. Mars is in this favourable position, of opposition to the sun, and as near as he can be to the earth's aphelion and his own perihelion, nearly every 8 years. It was so in 1862, soon after the first suspicion was entertained that the old distance of the sun and planets was overrated; and advantage was taken of Mars's position then to get his parallax by numerous observations at places as far apart as possible near the same meridians, such as Greenwich and Melbourne, Pulkowa and the Cape of

Good Hope, &c. And the result was that all the observations concurred in giving that parallax for Mars, which made the sun's distance nearly 92,000,000 miles, by the proportions known independently, as stated at p. 85. Another opportunity will occur in 1877, still more favourable than that of 1862.

I there promised to explain how the proportionate distances are found independently of the actual distances of the sun and planets. The proportion of sun and moon's distances was the earliest attempted, by the old astronomers, but with very erroneous results. If it were possible to observe the time of exact half moon, or when the sun and earth are at right angles to the moon, it could be done; because it is easy to measure the angle of their apparent distance from each other, and then the proportion between ⊖ ⊙ and ⊙ ☾ is found by a very simple piece of trigonometry. But unfortunately the exact half moon cannot be known by inspection, nor even by calculation without first knowing the sun's distance. If you look at apparent half moon through a telescope, or in a good photograph, such as those in Mr. Proctor's book on the Moon, or Nasmyth and Carpenter's, you will see that the better the means of observation are the worse the 'terminator' of light and darkness is in 'definition,' being in fact a very rough and gradually shaded boundary instead of the sharp straight edge which it looks to the naked eye. This method therefore, which was invented by Aristarchus already mentioned (p. 42), gave a result ridiculously wrong, making the sun's distance only 19 times the moon's. Another attempt by Hipparchus to measure the sun's distance by the time the moon

takes to go through the earth's shadow in a lunar eclipse, was and must be an equal failure practically, though it is theoretically right.

The proportion between the distances of the sun and Venus or Mercury is easily found from their greatest 'elongation,' i.e. their greatest apparent distance from the sun; which is when lines from the planet to the earth and sun are at right angles, corresponding to half moon: only here we have the greatest elongation to tell us when they are in that position. In that right-angled triangle then all the angles are known, because two are, and that gives the proportions of all the sides, though it tells nothing of their actual size. The exterior planets have no greatest elongation, except that of being apparently at an infinite distance from the sun at opposition. But as their periods are known, and therefore the proportion of their orbit gone through in a day, that can be compared with the daily portion of the sun's apparent orbit, and thus their distances can be compared. Or still better, now that we know from the law of gravity that the cubes of the distances of all the planets must vary as the squares of their periods, we can fix their proportionate distances at once. But they were found by observation and geometry long before that rule was known.

The lunar equation.—The sun's distance however can be deduced from the moon's when we also know her mass to be ·0123 of the earth's (p. 173). For the earth revolves every month round the c. g. of the earth and moon, which is 2895 miles from the earth's centre, as we found at p. 125. Consequently the earth is every fortnight alternately 2895 miles before and

behind its mean place, and would appear to people in the sun to oscillate about its mean place through twice the angle of 2895 ÷ sun's distance, instead of going uniformly. The sun does so appear to us to oscillate, and that is called the lunar equation, which is only another kind of parallax, or apparent change of place in a fixed body by reason of actual change in the moving one. That angle (measured from the mean position) is 6″·5 or ·000315, which = 2895 ÷ 92,000,000, and that last figure is therefore the sun's distance. But unfortunately it is impossible to observe the position of such a large and bright body as the sun with anything like the same accuracy as a star, and therefore this method is not capable of giving very accurate results. It may strike you as a further objection to it that the sun's distance is involved in calculating the moon's mass, from the tides and from nutation, as we saw at pp. 75, 173. But still this is not reasoning in a circle, because those calculations involve the cube of the sun's distance ÷ his mass, and that is a fixed proportion, represented by the length of the year, whatever they may be separately.

These various methods, and those mentioned at p. 85, and some others, all gave a parallax between 8″·87 and 8″·943. The latter was generally adopted for a few years after 1862, but was suspected to be too large even before the late transit of Venus, and now there seems no doubt that the smaller one is nearer the truth. If you think that this looks a considerable difference, you may realise what it is by the fact that it is equal to the width of a shilling seen 44 miles off. And now we proceed to what is considered the most perfect,

though the rarest, of all the modes of finding the sun's distance, by a

TRANSIT OF VENUS.

This phænomenon can only happen twice in 121½ years on the average, the two occasions being always 8 years apart—if there are two, for occasionally one of the pair misses. Those of the last century were on 6 June 1761 and 3 June 1769, and the two of this century are on 8 Dec. 1874 and 6 Dec. 1882; and there are no more till June 2004 and 2012.

This peculiar order of recurrence of the transits affects the manner in which it is usually (though not always) necessary to observe them, and therefore it should be explained. Venus's orbit is inclined $3° 23'$ (or ·059) to the ecliptic, or that is her greatest heliocentric latitude (seen from the sun). But a transit cannot happen unless her geocentric latitude (seen from the earth) is less than the sun's apparent radius of $16'$. (You must remember that celestial latitude is measured from the ecliptic, not the equator extended to the heavens.) Her distance from us at inferior conjunction is ⅜ths of her distance from the sun; or more exactly, her sun's distance is 2·6 times her distance from us; and therefore her heliocentric latitude must be within $16' \div 2{\cdot}6$ or $6'\ 9''$ for a transit to be seen; and her heliocentric longitude or distance from the node, must be within $6'\ 9'' \div {\cdot}059$ or $103'$. Indeed it must be rather less; for Venus's apparent diameter is then $39''$, or above a 60th of the sun's, and a mere grazing of the sun's edge would hardly serve for a transit. So we may say practically that the transit limits are $1° 40'$ on

each side of a node or 3° 20′ altogether. Therefore while the earth is within that heliocentric angle we may see the transit, other things being favourable; and as we go 59′ a day, we may call the transit limits either $3\frac{1}{3}$ days or degrees as we please. At present we pass through Venus's ascending node ☊, where she rises from south to north of the ecliptic, on Dec. 7, and the descending node ☋ on June 6. But they advance about 1° every 121 years in longitude from ♈, though they recede 41′ sidereally; but ♈ recedes 101′ by procession in that time. Therefore the transits of each century come a day later than the last.

A synodical period (p. 137) of Venus and the earth is 583·92 days, and the earth in that time goes once round the sun sidereally + 215°½, and Venus twice round + 215°½; which is only ½° short of ⅗ths of 360°. Therefore the conjunctions may be said to fall on the spokes of a great five-spoked wheel, in the order 142531 (making the figure called a pentagram), which wheel itself revolves backwards nearly 18′ a year; so that every fifth conjunction, in every eighth year, falls on the same spoke, but the spoke has moved 142′ backwards. Also when spoke 1 is at ☊ spoke 4 will be 36° in advance of ☋, and will work back to it in 121½ years, near enough for a transit. But the period between two transits (not of the same pair at one node) must evidently be some number of years divisible by 8, + half a year, because the conjunction must be at the opposite node. Therefore although 121½ years is the mean interval between each transit season, it may be either 8 years more or less; and that depends on the place in the earth's orbit where the transits happen, on

account of its different velocity at different parts. But it is unnecessary here to go into those minute details, which you will find in Mr. Proctor's book specially devoted to this subject, beyond saying that the December or perihelion transit season comes 113½ years after the June one, and that is 129½ after the December one.

It is necessary however to explain the conditions for a pair of transits. As the transit limits are 103′ on each side of the node, and the eight-yearly conjunctions are 142′ apart, it follows that if there is a transit more than 39′ beyond the node in longitude, there is room for another before the node 8 years afterwards; but if a transit is nearer the node than 39′ there can be no other at that season. So that a very central transit is necessarily a single one. Further we see that the first of a pair of December transits must be a northern one; as it must be after ☊, or over the northern half of the sun, and the second a southern one; and the first of a pair of June transits must be southern, and the second northern: the importance of which we shall learn presently.

Though Horrocks, with his friend Crabtree, first observed a transit of Venus, on 4 Dec. (N.S.) 1639, and promised a treatise on finding the parallax thereby, he did not live to write it. That was left for another young astronomer to do nearly a century later, viz.: the celebrated Edmund Halley, afterwards Astronomer Royal. It is true that James Gregory in his *Optica Promota* in 1633 had proposed a method which some writers have called equivalent to Halley's; and I followed their authority in the early editions, until I

saw Gregory's book and found his method to be entirely different, and useless: indeed he admits himself that it would be difficult. Halley's paper is in Latin in the Phil. Trans. of 1716; and it is singular that the method there proposed is really not that which is always called by his name, though they agree in using the difference of durations of a transit as seen by the parts of the earth farthest north and south of the ecliptic at the time: but Halley contemplated a very central transit, and his method, or that called his, is best for a very eccentric one, and generally (but not always) best for the second of a pair. This last, of Dec. 1874, was a striking exception to that rule; and that of 1882 will be also, because there are unfortunately no accessible antarctic places suitable for Halleyian observations of it. In 1769 his method was used, but it was unsuitable for 1761.

The principle of it is this. The people at the north of the earth (from the ecliptic) see Venus projected rather lower on the sun than those in the south; and therefore they see a northern transit lengthened, and a southern one shortened, and vice versâ; neglecting the earth's rotation for the present. The difference of the lengths of the two transit paths is accurately represented by the difference of the times they take, and the difference in lengths gives the distance between the two paths by very easy geometry. This requires no very exact knowledge of the longitude of the places, nor of the local time, but only the duration, which will be given by a clock that goes decently for a few hours. But we also want to know the time that a perfectly central transit would take, and luckily we can find that without knowing the sun's real distance

or diameter. For Venus with her shadow sweeps round him at the rate of 96′ a day, and the earth 59′, and so the shadow would appear to people on the sun to pass the earth at the rate of 37′ a day. But the same linear motion of Venus which appears 37′ at the sun appears 2·6 times as much here, or 96′ (another merely accidental coincidence with the other 96′). The sun's apparent diameter is exactly a third of 96′ (not that a transit can happen with the sun at mean distance, as they must be in June or December), and therefore a central transit would take a third of a day or 8 hours. It is very easy to calculate the distance between two parallel chords representing the number of hours of observed duration in a circle whose diameter is 8 hours. Suppose that distance is found to be = a 48th of the diameter; then if the places of observation are at the distance of 6795 miles across the ecliptic* the distance of the transit paths must be 2·6 times that, or 17,869, and consequently the sun's diameter 48 times that, or 857,716; and his distance we know is 108¼ times his diameter, or 92 million miles.

The transit paths can now be photographed, by a series of observations of Venus as a travelling spot, and the distance between them measured afterwards by comparison of the photographs magnified. Indeed a very few, or even one, observation of the transit when nearest its middle at each place will do, if they are properly selected, in the way which Mr. Proctor suggested, so as to make errors of centrality affect the result least; i.e. they should lie in a plane through the sun's centre. Many photographs were taken of

* I say the ecliptic for simplicity, but the real plane is that of Venus's apparent motion for the time.

the last transit, and those results are probably as valuable as any. At any rate they have the great advantage of being free from all 'personal errors' of observation: for it is well known that different persons' senses observe the same phaenomenon with different rapidity, especially when it is not a very distinct one, like Venus entering or leaving contact with the sun. Direct measuring of the distance of Venus from the sun's centre is impossible, because the sun's centre is not marked for us, and it cannot be determined very accurately from the bright circumference.

It is evidently important to select such places as will make the difference of duration of the two transits seen as great as possible, so that any errors of observation may affect the result as little as possible. Either the northern or the southern station must be in its winter, with all the risk of clouds and fogs. Fortunately however it is seldom necessary to go so near the pole which is in winter as to the one that is in summer; for the illuminated or transit-seeing hemisphere includes the pole which is leaning towards the sun, or is in its summer, but does not come near the other. Still there is great risk of fogs even in summer within or near the arctic circles; and so a good many places must be chosen to improve the chances of success. Moreover the earth's rotation complicates the problem, and sometimes helps and sometimes hinders the difference of duration. And again, that effect of rotation may be overbalanced by other circumstances, according to the direction in which the transit shadow sweeps over the earth: which we shall find was a very important consideration in the late transit.

Let us first take a southern transit over the sun in December, which we saw at p. 256 must be the second of a pair. The south pole is then turned towards the sun; so that places behind that pole, as we may call them, can see the sun and the transit, because they are then in the condition of having no night. And even beyond the antarctic circle for some distance they may see the beginning and the end of the transit, though they have their short night in the middle of it; but the beginning and the end are enough for measuring the time. Now the people in the south see the longest path of a southern transit on the sun, independently of rotation; and the effect of rotation is to carry those at the back of that pole, who are then going by rotation the same way as the earth in its orbit, along with the shadow of Venus over the earth, though not so fast as the shadow goes, and so it makes their transit longer still, and increases the difference between that and the one seen from the north.

But in the first of a pair of transits all that is reversed, and the rotation diminishes the difference of duration to the people at the back of the pole compared with that in the other hemisphere, and therefore such observations are of no use; although that may be compensated in another way in a very eccentric transit as we shall explain presently. Accordingly it was found, as 1761 approached and the position of Venus's nodes was ascertained, that the duration method would not do; and another was invented by Delisle, whose name has been similarly attached to it. It may be roughly described as taking the east and west parallax of Venus instead of the north and south. We

have now no more to do with transit paths on the sun, but the beginning of the transit is observed as an independent phænomenon from somewhere on the east side of the earth for the time, where it is seen earliest, and somewhere on the west side, where the transit begins some minutes later. The end of the transit may be observed as another independent phænomenon 4 or 5 hours afterwards from two other places near the eastern and western edges of the earth then. This looks very simple, but it has the disadvantage of requiring the longitudes of the places of observation to be known far more accurately than Halley's, and also the exact local time of the beginning or end of the transit, which may not be found accurately unless the weather has been fine very recently.

Delisle's method has to be applied thus. Venus with her shadow sweeps round the sun 96′ a day, and the earth 59′; and therefore the shadow would be seen from the sun to overtake and pass over the earth at the rate of 37′ a day or 1″·54 a minute. Suppose that in a central transit (for simplicity) the 'ingress' begins 11 minutes earlier at the extreme east of the earth or sunrise, than at the extreme west or sunset. Not that observations so close to the horizon will really do, because they are distorted by the refraction of the air; and we must be content with places less than the earth's diameter apart; but the proportions would remain the same. That makes the earth's diameter 11 × 1″·54 or 17″·7, as seen from the sun, which is only another expression for ·000086 of the sun's distance (p. 9); which is therefore the earth's diameter divided by that decimal, or about 92,000,000 miles. But you

see the 11 minutes' difference is taken at two places right across the earth, and we must know their longitudes exactly, as well as the two local times, to get the true differences of time and position. An error of only a second might make that observation worse than useless for correcting our present almost certain knowledge of the sun's distance. Accordingly Delisle's method has always been regarded as the second best, and decidedly inferior to Halley's whenever that is suitable: in fact I know of no book on astronomy where it was even described, until lately in the discussion which I must advert to presently. The calculations for an eccentric transit are of course less simple, but it is not necessary to explain them here.

The Delisle observations of 1761 differed so much from each other that they could not be relied on; for though some of them turn out now to have been right, it was impossible to know then which they were. The sun's distance had been estimated before that, by other methods, at about 90 million miles (see p. 85), and so it remained undisturbed by the results of the 1761 transit, until the unlucky mistake in the deductions from the observations of the 1769 transit, which was suspected in 1854 and had become all but certain before 1857; or more probably in some of the observations. None of the proposed explanations of the mistake are now considered satisfactory: so I do not repeat them.

The phases of a transit which are best for all except photographic observation are the first and last internal contacts of the black spot of Venus with the bright disc of the sun; and these are shortly called ingress, which means completed ingress, and egress, which means

egress just beginning. But herein lies the difficulty. Apparent internal contact is not real, on account of that quality called *irradiation*, which makes a bright body always apparently encroach on a dark one, so as to make the bright one larger, and the dark one smaller, and therefore diminishes the size of Venus seen against the sun, while the sun himself is enlarged (p. 117). But until the real Venus has got within the real sun she hangs on to his edge, by a 'drop,' or 'ligament,' which gets thinner and thinner until it becomes a fine line and breaks. In like manner egress begins with such a line, which thickens until the black spot appears to be just at internal contact with the sun. At any rate this appears so to some observers and with some telescopes; but the late transit was less affected by this difficulty than was expected from previous experience. This hanging on varies as to time with the obliqueness of ingress and egress, which depends on the distance of the transit from the sun's centre; but in an average transit it lasts somewhere about 20 seconds; in a central one it is less. It also varies with the state of the atmosphere and the character of the telescope, lasting several seconds longer when the sun is bright than when it is dim, though of course it cannot be actually cloudy for a transit to be seen at all: so that unfortunately the real internal contact cannot be identified either with the breaking of the drop or any other phase of it. Even where there is no 'black drop,' apparent contact is not identical with real, for the irradiation exists still; but the difference may be constant, and therefore immaterial.

The controversy between Sir G. Airy and Mr.

Proctor as to the best mode of observing the 1874 transit is as much a part of the history of astronomy as the proceedings in the discovery of Uranus, and the conditions of the problem cannot be understood without some account of it.

Sir G. Airy told the R. A. S. in a paper of 1857, and still more strongly in another of Dec. 1868, that Halley's method would be inapplicable to the 1874 transit, because it is the first of a pair, and so no advantage can be taken of the earth's rotation, as I have explained already. For the same reason he said it would be applicable in 1882, and suggested Sabrina-land as an antarctic station for it, in another paper of June 1864. In 1868 he expressed some doubt whether that would be suitable in point of climate, but strongly urged that an exploring expedition should be sent to ascertain that point. Several Admiralty and other naval authorities at once supported him, and said there was no insuperable difficulty in such expeditions, and expressed full confidence in its being undertaken. Representations to the Admiralty were duly made, and it was understood and stated that the expeditions were to go; but that the 1874 transit was to be observed by Delisle's method only, as the A. R. said Halley's was unsuitable.

But a month after the publication of that paper of 1868, viz., in March 1869, Mr. Proctor, then a young man much less known than now, astonished the R. A. S. by announcing that he had found the Astronomer Royal's conclusion that Halley's method 'fails totally for 1874,' was itself totally erroneous; and that on the contrary that transit would be on the whole singularly well adapted for it, giving larger differences of duration

than that of 1769, or any other of which the elements are yet known. He further said that sending antarctic expeditions for Halleyian observations in 1882 would be a perfectly useless waste of money and risk of sailors' lives, because the sun will be too near the horizon for any reliable observations at the places proposed, and no other suitable ones would be accessible. M. Puiseux, of the French Board of Longitude, also disputed Sir G. Airy's conclusions as to 1874, but not so decidedly.

In a further paper Mr. Proctor gave the mathematical proof of his conclusion about the 1874 transit. It was questioned on one point by Mr. Stone, which he answered, and his accuracy was no further questioned, either by Mr. Stone or any other mathematician in England or elsewhere. No public notice however was taken of this striking contradiction, beyond dropping the projected antarctic expeditions for the 1882 transit. As 1874 drew near, Mr. Proctor began to write again upon the subject, but still without the least notice from Sir G. Airy, until it was proposed and carried in the Council of the R. A. S. to give the Society's medal to Mr. Proctor for this and his other labours. Then he wrote a letter deprecating the confirmation of the award, and depreciating Mr. Proctor's work as much as possible, but still not disputing its accuracy, nor explaining how it could be unimportant unless it was wrong. Nevertheless the award was confirmed by a larger majority, of 2 to 1, though not 3 to 1 which the bye-laws require.

Shortly after this there were articles in the 'Spectator' and the 'Times' calling public attention to the

risk the nation was running of a more discreditable miscarriage than that of 1769, by wilfully neglecting what every astonomer had considered the best mode of observation, provided it is applicable. Mr. Proctor afterwards avowed the authorship of the former of these, and the latter was guessed by many persons to be mine. Neither of us had any idea that the other was writing at the same time. Thereupon the Admiralty did actually move so far as to ask Sir G. Airy for an answer to these articles; and he wrote one and read it to the R. A. S. in March 1873. He no longer maintained the inapplicability of Halley's method, but argued that it would be on the whole inferior to Delisle's; and took up an entirely new ground, of climatical objection to the best stations for astronomical results both in the extreme north and south; especially the proposed Siberian ones, which he did not believe Russia would undertake with the very small probability (as he assumed) of fine weather there in December. It turned out however that both the probability and the fact were the other way, and most valuable observations were got there. He also quoted Dr. Oppolzer of Berlin for the not very novel remark that longitudes for Delisle's method can be found more accurately now than in 1761, when it practically failed. But the risk of error in the local time remains.

Mr. Proctor immediately replied, with a table of numerical results for the combination of every pair of stations in the world which had been suggested, which showed a considerable superiority in Halley's method in several of them. Not that it really signified whether Delisle's were as good as Halley's, as he had never

proposed to abandon Delisle's, but only not to abandon Halley's, or north and south observations by photography, using each at the best places for them. As to the climatical objection for the south, he merely cited the naval opinions which had been so promptly given in favour of antarctic expeditions when Sir G. Airy wanted them. One of those naval authorities indeed, the then Admiralty Hydrographer, wrote to the 'Times' to say, in short, that he had changed his mind, and now agreed with the A. R. in condemning an antarctic expedition; as he had agreed with him in 1868 in recommending it: a truly valuable opinion.

The Admiralty—i. e. Mr. Goschen the First Lord thereof—thought they had done quite enough by consulting the Astronomer Royal as to whether the Astronomer Royal was right or wrong, and told Parliament that they meant to follow his advice, and have no Halleyian observations. They had been furnished with Mr. Proctor's reply; but it is not the nature of official people to regard such things, even to the extent of referring the matter to some independent authority, as had been urged in the newspapers.

But by this time astronomers in general had begun to be alarmed; and after some unpublished correspondence (which I had not heard of when I wrote a summary of these proceedings in the 'Times' of 2 Jan. 1874, in a letter with my name—none of the later articles were mine, or Mr. Proctor's, except letters with his name) a resolution was moved by Professor Adams and unanimously carried at the Greenwich Visitation in June 1873, that an 'application be made to the Government for the means of organising parties of observers

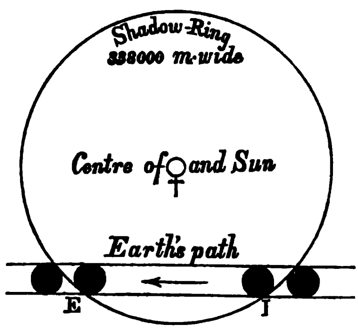
in the Southern Ocean with a view to finding additional localities for observing the whole duration of the transit,' i. e. for Halleyian observations.* This was rather too weighty a communication for the Admiralty to extinguish with the dictum of their Astronomer Royal, and they agreed to send the required expedition, notwithstanding their Hydrographer's apprehensions, which it is gratifying to find were not realized. Halleyian observations of the transit were also at last settled to be made at several places both by us and other nations. Indeed other nations, and notably America, had rejected Sir G. Airy's views for Mr. Proctor's long before. Mr. Proctor publicly expressed his satisfaction at this result, and also that some Indian observations were to be made, which he had proposed and the A. R. had expressly rejected, both in his letter to the Admiralty and before. Nevertheless in a letter to the R. A. S. Council he denied that any alteration had been made in his plans: which statement, with the whole matter, I leave to the reader's reflections, as I did the history of Neptune.

It is however necessary to explain how his original mistake came to be made; or rather, how it happens that Halley's method can be the best for a transit in which no advantage can be taken of the earth's rotation. The answer is the usual one, that the mistake arose, not from miscalculation, but from not taking due account of all the circumstances; i. e. in this case, of the peculiar manner in which this transit sweeps over the earth on account of its great eccentricity on the sun; for Venus

* In the R. A. S. 'Notices' it is incorrectly added that 'the A. R. made this application at the suggestion of the Board of Visitors.' It was made by the President of the R. A. S., as chairman of the Board.

would be nowhere nearer to the sun's centre than 7-8ths of his radius. We must consider then how the earth enters and emerges from that imaginary ring, or rather, conical shell in the heavens, where the outer edge of Venus just hides the circumference of the sun; which we may shortly call the shadow-ring at the distance of the earth. The *whole* of Venus would cast a ring of shadow from the sun's circumference, considered as a mere bright ring, about 2½ as thick as her own diameter; but as we only want internal contacts, the ring we have to deal with is a mere circle of no thickness, but with a diameter about ⅞ of the sun's, or 42 times the earth's diameter.

We may treat Venus, Sun, and shadow-ring as stationary, if we give all the relative motion to the earth, which will then go through the ring as in this figure, looking at the sun. For making maps of the earth during the transit, such as Mr. Proctor's, it is necessary to look *from* the sun, and then of course the direction is reversed.



The earth goes as much below the centre of the shadow-ring as Venus goes above the sun's centre, and I have exaggerated the size of the earth in the four small circles at I (ingress) and E (egress) to show more clearly that the upper left hand or north-eastern parts of the earth for the time first enter the ring,

or see 'ingress most accelerated by parallax,' and the south-western parts enter last, or see it most retarded.

The ring is so large compared with the earth that the piece of it which sweeps over the earth is almost a straight line; and as the earth only takes some 25 minutes to pass through it, the latest ingress is seen by nearly the antipodes of those who saw the earliest. I do not represent the apparent obliquity of the path of Venus over the sun, which is usually and rightly given in pictures of the transit, because it has nothing to do with the obliqueness of entrance and emergence on the circumference, which depend solely on the eccentricity of the transit. That apparent inclination of the path is due to the inclination of Venus's orbit to the ecliptic, which is exaggerated by her apparent daily motion in longitude being reduced by the earth's motion from 96′ to 37′, while the latitude is not reduced, and also due to the slight inclination of the earth's axis to the right at that time, which would be none if it were exactly at the solstice. Her latitude and longitude are equally affected by distance. This picture will agree with the usual ones if you incline the page nearly 15° to the right.

At egress E, the converse of all this takes place, and the earth having turned some way round in the 3½ hours which its centre takes to cross that chord of the ring, the first places that see egress are those on the south-east side of the earth at that time; and their north-west antipodes (nearly) see the latest egress. Then the only proper way to select places of observation is to make two maps of the earth at (mean) ingress and egress, and to find by trial what places in the north

combine the earliest ingress with the latest egress and what places in the south combine the latest ingress with the earliest egress; and all pairs of such places will give the best results for the Halleyian method. By this process rotation and everything else has been taken into account in a way that is impossible by any general or *à priori* reasoning.

Mr. Proctor shows that between properly selected places the combined difference in 1874 was 33 minutes, whereas it was only 24 in 1769. So it turns out that this was a singularly favourable transit for Halley's method, its great distance from centrality preponderating over the loss of any advantage from the earth's rotation, which cannot benefit the first transit of a pair. In 1882 unfortunately the double advantage cannot be gained, for the reason given at p. 257.

The second edition of Mr. Proctor's 'Transits of Venus' contains a summary of the disappointments and successes of the late observations, and on the whole the successes appear to have been as numerous as could fairly be expected under the uncertainties of weather. It appears that the Delisle observations were the most unlucky of the three kinds I have mentioned. And though the Astronomer Royal is not responsible for the weather, it would have been an aggravation of our failure and 'international' disgrace if we had persisted in taking no observations except those which the weather happened in a great measure to defeat. The Halleyian and mid-transit photographic observations on the other hand are said to have 'given a series of excellent results.' Among the latter should be especially noticed those obtained by Lord Lindsay in an expedition to the

Mauritius fitted out by himself at great expense, as he had previously done for the eclipses of 1870 and 1871. One of the many curious phænomena of this transit history was the Astronomer Royal's refusal, in his answer to the Government and the newspapers, to recognize Lord Lindsay's expedition as being anything more than a hope that a private astronomer would do something equivalent to what he had advised the Government to do, so as to set free a ship for employment elsewhere. As a deduction however from these last successes, Mr. Proctor mentions the strange omission to provide for mid-transit observations at the best place for them in the southern hemisphere, Cape Town, though we have a most able local astronomer royal and an observatory established there.

I have already given throughout the book the conclusion as to the sun's distance, so far as the results of the transit have been yet worked out. It is a little unsatisfactory that they do not quite agree with the figures which had been adopted from other methods in the last ten years; and the observations of Mars in 1877 are looked for with the more anxiety, as well as the remaining transit of 1882.

Transits of Mercury are much more common, occurring at intervals of 7 and 13 years; but the same cause which makes them more common makes them useless for finding the sun's distance, viz. Mercury being twice as near the sun as Venus is, and twice as far from us; which makes the distance between the transit paths four times less than for Venus, and much more than four times less likely to give accurate results for the sun's diameter or distance.

All astronomical observations, except those for finding a parallax, have to be corrected for the earth's parallax, or as it is called shortly—parallax, or the distance of the observers from the earth's centre. Otherwise the observations made and recorded at one observatory would be unintelligible and useless at any other. By that correction they are 'reduced' to the earth's centre as a common point for all observations alike. We have now to consider two other corrections which have to be made before any observation can be considered complete, and in a state fit to be recorded for future use.

ABERRATION AND REFRACTION.

The stars and sun and planets are apparently displaced, or are seen in wrong places, from another cause, called *aberration*. The discovery of it by Bradley in 1727, the year that Newton died, was a consequence and confirmation of the previous discovery, that light takes a definite and measurable time to come from the sun and planets; not that the planets have any light of their own, but only reflect the sun's, as the moon does, subject to what is said of Jupiter and Saturn at p. 219. We may explain aberration thus:—If you are running when the rain comes down straight without any wind, you get wet in front and not behind, and the rain beats against you as it would if you were standing still and the wind blowing in your face. And if you carry an empty telescope tube pointed straight up, the rain will not fall through it, but will strike against the back inside: if you want the rain to fall through, you must slope the tube forwards, more or less according to your

velocity forwards compared with that of the rain downwards. Then for rain substitute light, and the motion of the earth for your own running, and you know what aberration is.

Therefore whenever we are moving directly across the light from any star, it appears before its true place by 20″, which = ·00001 (see p. 9), and is the proportion of the velocity of the earth to that of light (p. 85). 'Before' means in the direction the earth is going; and if you look at p. 235 you will see that that moves ⊙ a little to the right, and so diminishes the angle S*e*⊙, or E⊙*e*, or earth's heliocentric longitude, or sun's geocentric longitude, by 20″ always. The earth would only be seen by any star in the ecliptic to oscillate half-yearly; and therefore such stars move backwards and forwards 40″ by aberration every year. But stars near the poles of the ecliptic would see the whole of the earth's orbit as a circle (neglecting its insignificant ellipticity), and so they describe a circle of aberration 40″ in diameter. All other stars have their aberration circle foreshortened into a perspective ellipse, with a major axis of 40″ lying across the line to the earth, and the minor diminished according to the star's latitude or distance from the ecliptic, where the ellipse sinks into the line of oscillation.

This aberration or annual revolution of all the stars in various little orbits of their own, according to their distance from the ecliptic, affords the only *direct* proof that the earth goes round the sun, and not the sun round the earth. It was absurd enough to suppose, as the ancients did, that all the stars go round the earth daily as if they were fixed in a frame with its axis

through the poles of the earth; but it is utterly inconceivable that they should all have such peculiar individual orbits besides, as would be necessary to produce the visible effects of aberration with the earth stationary. This motion however is too small to have been visible before the days of telescopes.

Refraction.—There is yet another correction to be applied to most telescopic observations before they can be said to have given the true place of a star, as it would appear from an earth no bigger than a point, quite still, and without an atmosphere to bend the rays out of a straight line. For this is what the atmosphere does, and it is called *refraction.* If you hold a straight stick any way but upright in a trough of water, it appears to be suddenly bent upwards, or farther from upright, with an elbow at the surface of the water, and the trough itself looks less deep than it does when it is empty; so that if you stand where you cannot quite see the bottom of the trough empty, you will be able to see it when it is full. And conversely, if your head were under water all things outside of it would appear elevated.

The reason is, first, that you see everything in the direction of the rays as they at last reach the eye, by whatever road they have come, as things seen in a mirror appear to be behind it; and secondly, the rays of light are always bent towards the perpendicular in the denser of two mediums which they pass through obliquely. Consequently anything in the water is seen by rays in the air more oblique than those which started from it in the water, and so it appears lifted. Again, as air is denser than empty space, though very much thinner

than water, the rays which come to us from the stars are bent towards the perpendicular, and so they appear higher than they are, except when they are already as high as possible, or in the zenith. And the sun and moon always do, since they are never in our zenith; and the lower they are the more they are raised by refraction. For the law of refraction is that the distance of any point of a ray from the perpendicular to the surface, in the second of the two given transparent mediums, always bears a fixed proportion to what the distance of the same point would have been if the ray had not been bent aside or refracted. This is shortly expressed to those who know a little trigonometry by saying that the *sine* of the angle of incidence bears a fixed proportion to the sine of the angle of refraction, depending on the nature of the two mediums.

Besides this, the density, and therefore the refraction of the air decreases upwards, as you may see by taking a portable barometer up a mountain; so much that, although an atmosphere of uniform density and of the known weight of 15 lbs. on the square inch would only reach 5 miles high, it does in fact reach about 80. Consequently the rays are not bent with a single elbow, like the image of the stick in water, but into a curve, continually getting more upright as they approach the earth, and you see the star in the direction in which the rays at last reach the eye. Moreover the lower the star is, the more obliquely the rays enter the air, and the more they are bent. Refraction is also diminished by heat, as that expands the air and makes it thinner; and it increases with a rise of the barometer, which indicates increased density in the air, and so it can

only be found and applied by tables which have been prepared from long experience. Sir G. Airy calls 'refraction the bane of astronomers,' because it cannot be calculated with certainty, like aberration and parallax.

The average amount of it for objects half way up from the horizon to the zenith is about 1′, but at the horizon it is as much as 33′, which is rather more than the apparent diameter of the sun or moon. Consequently they have really set, or have not risen, when they appear to be just above the horizon, being lifted their whole height by refraction; and the moon may be totally eclipsed with the sun apparently above the horizon, though the earth is really straight between them. From the same cause the French cliffs, and even ships on the sea near them, can be seen from the English coast in some states of the atmosphere. There is no refraction sideways, and it increases so rapidly towards the horizon, that the lower edge (or *limb*, as they call it) of the sun or moon is lifted rather more than the upper, and therefore they do not appear quite round, but visibly broader than high when they are just rising or setting, and yet not exactly elliptical, but more flattened at the bottom than the top.

Their appearing larger at the horizon to the naked eye is only an optical delusion; for in fact they appear smaller, as the vertical diameter is diminished by refraction and the horizontal one is not increased. The delusion is generally attributed to our being able to compare them at the horizon with things on the earth. But I doubt if this is the proper explanation; for it is equally the case when it is too dark to see anything

but the moon itself just rising, and at sea, where there is nothing else to be seen. You may observe too that a man looks much larger against the horizon on the top of a hill than when you are at the top and he is at the bottom. I believe the reason is that when things are on the horizon we compare their linear dimensions with the length of horizon which the eye takes in, but that in the middle of the sky or earth we compare the area they cover with the area the eye takes in. Assuming the eye to see distinctly over 30° of apparent width, the moon on the horizon covers one 60th of that; but when it is high up it only fills the 3600th of the area of sky which the eye sees all around it. Between these two extremes some compromise is made unconsciously, and a different one by different people's eyes.

Another theory has been propounded, that the enlargement of the sun and moon on the horizon is due to a peculiar effect of the red rays on the eye, which then evidently preponderate, from the greater absorption of the other colours by the atmosphere. But if so, the red sun in a London fog ought to look larger than the white sun in a damp fog: which it does not.

From another cause the moon really measures less when she is rising or setting; that is, when we are just coming into or going out of sight of her. For the parts of the earth to which the moon is just rising or setting may be farther off by half the earth's diameter than the places full in front, which have the moon on their meridian; and the earth's radius is one 60th of the moon's distance. Therefore her diameter will appear one 60th less, and her whole disc about one 30th less to

the places farthest off, which can just see her, than to the places nearest. But the difference of the earth's radius is practically nothing in the distance of the sun, and therefore you cannot say that the sun appears smaller on the horizon than on the meridian, as the moon does, except from refraction.

Twilight is also caused by the air, but not in the same way as refraction. When the sun is not more than about 15° below the horizon his rays are a little *reflected* down to us from the vapours and other small particles of matter in the air. The more obliquely the sun goes down the longer he takes to get as low as 15° below the horizon, and consequently twilight lasts longer in high latitudes than within the tropics, where the sun's path may be vertical or through the zenith at noon, and is quite so in some latitude there every day.

The blueness of the sky is also due to the reflection of the violet rays by the air, from some unknown cause; and is therefore greater when the sun is low, and greatest at night, when he is below the horizon altogether. Professor Tyndall has shown that the blue rays are reflected by the very small particles of various gases which the other rays pass through. So, he adds, the crimson glow of the Alps in the evening and morning is due to transmitted light which has its blue constituents sifted out in passing through a great length of atmosphere. The same fact also accounts for the light of the sky being polarized (p. 103) by oblique reflection from the particles of air.*

* 'Proceedings of the Royal Institution,' v. 440.

METEORS, OR SHOOTING STARS, AND AEROLITES.

Until a few years ago nothing was known of these bodies except that they were occasionally seen in small numbers or singly; but twice a year, i.e. about the 10th of August and 14th of November, there was what was called a shower of them; and further, the November shower was found to be much thicker every 33 or 34 years. These phænomena had been observed from very early ages. The November shower, or at least a shower of shooting stars, is recorded in the Anglo-Saxon Chronicle of 704, and again in 902, and the period of the August shower is recorded in Chinese Annals of 1000 years ago as agreeing very exactly with a sidereal year; so that it comes a day later in 71 equinoctial or common years. That has no 33 year maximum, and the effect of it might be produced by a continuous elliptic orbit, of any great length, full of meteors going round the sun in a plane different from the ecliptic, and crossing the earth's orbit on Aug. 10; or else by a mere cluster of meteors revolving round the sun in exactly the earth's sidereal period, which would make the earth fall in with them always if it does once, until the eccentricities of the two orbits and the motion of their apses throw them out, as it certainly would have done in much less than the 1000 years; and therefore that hypothesis is very improbable. Indeed it is now considered certain that their orbit is a very long one, with aphelion beyond Neptune, but perihelion nearer than ours, and a period of 147 years for each meteor. As many as 100 regular meteoric days are now reckoned in the year, besides the stray meteors which are seen almost every

night, though there are no such large streams as the August or November ones.

For some time the theory of Professor Newton of America was received, that the November meteors revolve in a nearly circular orbit rather smaller than the earth's, inclined 17° to the ecliptic and crossing it at the earth's place on Nov. 14, and going the opposite way to the earth in 354·62 days; and that this orbit is a thin stream with a thick lump in it, which we therefore fall in with every 33 years, and a little of it the next year on account of the length of the lump. As a matter of fact the nodes of the meteoric orbit recede with reference to their direction, or advance in longitude with reference to ours, 52″·4 a year; and that, with the precession of our equinoxes, from which longitude is reckoned, makes the meteors come a day later nearly every 34 years (only an accidental coincidence with the other 33 or 34).

But Professor Adams added to his other discoveries the calculation that the nodes of such an orbit would not recede 52″ a year under the disturbances of the planets, but only 21″; and consequently that that is not the true one. And it is now universally agreed that the true orbit is that which he propounded, making the period of each meteor (for they must be considered individually under the law of gravity) the 32·25 years which have been the average period since 18 Oct. 902 (N. S.), with an eccentricity 54 times that of the earth's orbit and a major axis 10·34 times greater, and therefore reaching beyond Uranus at their aphelion, and crossing our orbit near the meteoric perihelion. Consequently the thickest part of this long elliptical ring crosses our

orbit once in the 33·25 years; and is estimated from the time the earth takes to go through it (about 2 hours), that it is 100,000 miles thick at that part. Moreover as we sometimes cross it two years running, it follows from the necessary velocity in such an orbit, that the thick part must be 1000 million miles long.

Moreover these two orbits have been identified with those of two known comets; and great astronomers in various parts of the world almost simultaneously adopted the theory of identifying comets with the meteoric orbits. But the constitution of both meteors and comets is still uncertain, or rather perhaps, variable, as the spectroscope is said to prove that some of them are gaseous, while many meteors, and of course all that reach the earth, are solid and composed of known substances, of which iron is the principal. The cause of their luminosity is simply the heat generated by their rushing through the air, which entirely burns up the small ones. It is stated in an article upon them in Mr. Proctor's 'Orbs Around Us,' that they appear at 72 miles high, the atmosphere then being just dense enough to heat them by friction, and disappear by combustion at 52 miles high: but this must depend somewhat on their size.

One of the things longest noticed in the great showers was that all the meteors appeared to radiate in all directions from one point in the sky, which is a star in the constellation Leo for the November ones, and they were sometimes called Leonides accordingly. But this is only the effect of perspective, arising from the way the earth is going just then, nearly the opposite way to the meteors, which doubles their relative velocity and

makes it about 40 miles a second. If a quantity of rockets were shot straight forward to us from the end of a street they would all appear to radiate outwards in every direction, except the few which came directly to us, which would have no perspective and appear stationary points; and so do some of the meteors. The 'radiant,' or point from which they radiate, is that to which the earth is driving, which is always 90° from the sun (disregarding eccentricity of our orbit) or in longitude 90° less than the sun's. It is of no use looking for the shower until our part of the earth has come by rotation into the position of facing it, which must evidently be near midnight; for then we are going by rotation in the same direction as by revolution, remembering that they are both from right to left in this hemisphere. And the shower begins about 15 minutes sooner in the south hemisphere than the north, because the south of the earth touches the inclined plane of the meteoric orbit first.

Meteors vary in size from a few grains' weight to about a pound, but few genuine meteors are above that. The larger bodies called aërolites sometimes reach a good many tons, though that is rare, and people have been killed by them. These mostly come by day, and the theory has lately been started and advocated by Mr. Proctor, that they are ejected by the sun, in which case they would almost certainly reach the earth by day, i. e. would strike the side of the earth for the time facing the sun. The nocturnal meteor systems are probably innumerable, as the earth encounters at least 100 of them, and as they are not particularly attached to the ecliptic, it is calculated that the

chances are in favour of there being considerably above a million of them. It was an old idea that they were ejected from volcanoes in the moon; but first of all there is no reason to believe that those volcanoes are alive now; and if they were, they would have no power to eject stones far enough to exchange the moon's attraction for the earth's unless they were enormously stronger than any volcanoes on the earth. There is no kind of evidence in favour of that notion. No elements foreign to the earth have been found in any meteors, though some of them come from the farthest regions of the solar system, and some probably from the sun.

We have already seen the important part which they are supposed to play in maintaining the sun's heat. Mr. Proctor suggests ('Other Worlds,' p. 210) that the planets too, and especially the large ones, are constantly gathering in meteors and so enlarging themselves and maintaining their own heat, and in fact may have been composed by aggregation of meteors. They may also be ejecting meteors, if the sun is.

Laplace's Nebular Theory should be noticed here as the rival one for the composition of the solar system: not that any theory can be invented for generating all the motions of the universe without some original impulse besides the mere creation of matter and attractive force and heat. It is quite impossible that all the 8 planets and the moons of all but two of them, and the 160 planetoids, can have come to revolve in the same direction and so nearly in the same plane without some common impulse at first. Laplace invented the following theory to account for it.

If an enormous globe of nebulous matter were once

set slowly whirling in the general direction, which we call from west to east, it would gradually contract by cooling, and revolve faster, because its 'moment of inertia' is thereby diminished, and then throw off rings, and then each ring might break and gather itself into a globe, which would partake of the original rotation besides revolving in the general direction; and each globe itself might afterwards throw off smaller rings, which would either stay as such, like Saturn's, or break and run together into moons.

The distances of the planets therefore represent the force of expansion of the original nebula against the attraction of its parts. Indeed whether this nebular theory is true or not, the distance of everything from everything else represents all the force which has been expended from 'the beginning' in separating them, or the vis viva they would acquire in coming together by attraction. And its increase as they approach is no 'creation of force;' nor its decrease as they recede, a 'destruction of force:' it only reappears in one case and becomes latent in the other, as explained at p. 25.

The explanation devised for the abnormal motions of the moons of Uranus and Neptune is that those planets had their axes turned much more than the earth's out of the perpendicular to their orbits before they threw off the rings which became their moons. But I have seen no suggestion to explain how the earth's axis could be so disturbed after it had become a rotating globe (which resists such disturbance) and thrown off a moon; or how the November meteors came to revolve in the opposite direction.

It must be added however, that experiments can be

made to exhibit the process, and to show that a mass of fluid, and therefore of nebulous matter, set rotating, will throw off rings when its velocity is increased enough to overcome its cohesion; and that if the ring breaks it will gather itself up into a globe by the attraction of its particles. The experiment is this: water is lightened with spirits of wine till it has such a specific gravity that oil will lie anywhere within it. The oil thereupon becomes a globe by its own attraction within itself (p. 11). The whole, inclosed in a glass box, is set spinning, with an axis through the oil to make it spin too. The oil globe first spreads into an oblate spheroid: then as it is whirled faster it throws off a ring, which revolves round it; after a time the ring breaks and gathers itself up into a smaller globe which rotates besides revolving round the large globe; and then another ring is thrown off as the velocity is again increased, and so on.

The experiment is defective in neither being able to represent the attraction nor the contraction of the globe, on which the whole theory depends. For the attraction of a shrinking globe on its own equator increases as the square of the radius decreases, and the centrifugal force (radius $\times$ angular velocity^2) may or may not increase faster than that, according to the variations of inside and outside density. English mathematicians seem to consider this theory too uncertain to be worth expounding; but I believe French ones have calculated from the present sun's rotation, that when it was as wide as each planet's orbit it would turn in nearly the present period of that planet: and the same of the planets and their moons.

And if the solar mass did turn at that rate, there would be the same equilibrium as now between the centrifugal and attractive forces; for the attraction on any planet, and therefore on the sun's equator, if it swelled into a globe as wide as the planet's orbit, would be the same as it is now, since a globe attracts at any given distance as if it were condensed into its centre; and we shall see afterwards that a nebulous globe of any uniform density can all rotate together. But if the density then increases inwards the 'moment of inertia' will decrease, and the outer parts will be dragged by attraction after the inner, which have begun to revolve faster, and so they will be made to go faster too, and their centrifugal force will make them fly farther out, increasing the oblateness of the globe, and perhaps separating as a ring.

It should be noticed also that an impulse given to a solid planet a little beyond its centre would produce both the present motions of rotation and translation, as you may see by striking a floating ball in that way.

Mr. Proctor suggests as an objection to the nebular theory, that it requires each planet, beginning with the farthest, to have been generated enormous ages before the inner ones; and there is nothing in their appearance to indicate that. And now that it is all but certain that Jupiter and Saturn are very hot instead of very cold, that fact tends the other way. But we have devoted space enough to all this, which as yet is mere matter of speculation, except that some of the infinity of meteors, running about in every direction within the solar system and beyond it, must be continually getting devoured by the planets and the sun.

THE FOUR CONIC SECTIONS.

There are two other curves besides circles and ellipses, in which the planets could, and some comets probably do move, under the law of gravity varying inversely as the square of the distance. They are the *parabola* and the *hyperbola*. A parabola is the curve described by a stone thrown into the air any way but straight up, or by water rushing out of a hole in the side of a pretty full cask, except that the resistance of the air spoils the accuracy of the curve. You may see a hyperbola in the shadow on the wall from a round shade over a lamp set near it. But the remarkable thing is that these four curves, the circle, the ellipse, the parabola, and the hyperbola, which are the only possible ones for the heavenly bodies to describe under the existing laws of nature, are all produced by cutting straight through a cone in different directions. You may have heard the term *conic sections* without knowing what it means, or why whole books of mathematics are written about them. There is no part of geometry with so many curious and elegant problems in it; and the reason of its importance is that all the heavenly motions are performed in conic sections. I must therefore explain what they are, as far as I can without mathematics.

A cone may be defined as a round pyramid, or a body with a circle for its bottom and a point for its top, and straight sides. When the top is vertically over the centre of the circle it is called a *right cone*, and when it is not, an *oblique* one. But cones are always assumed to be 'right' unless the contrary is specified,

and we shall deal with them so. If you cut a few cones out of some soft stuff such as a turnip, or get them turned in wood to be sawn through afterwards, it will give you a much better idea of the different conic sections than any pictures. First of all it requires no cutting to see that all sections parallel to the base of the cone, or at right angles to its axis, must be circles; and a circle may be considered an ellipse of no eccentricity, or with its two foci run together. Next, any other section of the cone, which comes out of both sides, is an ellipse, as you will see by cutting it through in that way. And I should tell you that the cone is considered to be extended to any length required for the elliptical section to come out again lower down on one side than the other. You would hardly believe without trying it, that the lower end of the section which comes out where the cone is broad is no broader than the higher end where it is narrow: yet so it is, on account of the difference of the angles made with the sides of the cone.

An ellipse is also produced by any oblique section of a cylinder, as a circle is the section directly across a cylinder. But in fact a cylinder is only a cone of infinite length, so that its sides may be considered parallel, as when we look at the stars, which are some of them much bigger than the sun, and yet look like points. This is the explanation of what I said at p. 49, that an ellipse is the oblique view of a circle, whether near or far off: if near, the lines of sight form an oblique cone; if very far off, they may be considered to form a thin cylinder. But the centre of the ellipse is not in the axis of the cone, though it is of the cylinder.

A parabola is a very different looking thing, though it is produced by a very little deviation from the section of a cone which makes an ellipse. It cannot be cut out of a cylinder. It is like half—or rather, some indefinite part less than half, of an ellipse of infinite length. It has no minor axis, for its centre is at an infinite distance, that is, nowhere. It is made by cutting through a cone anywhere by a cut parallel to the opposite 'slant side.' Therefore there can be only one parabola, as there is only one circle, at any given distance from the top of the cone, which must now be considered of infinite extent downwards. The two legs of a parabola are always getting more parallel to its axis, but never become quite so, however far they are extended.

A hyperbola is made by any section of the cone which is neither parallel to its slant side nor comes out of it at both sides.* It does not differ much in appearance from a parabola, only spreading out wider. The slightest deviation of the parabolic cut towards the vertical turns the parabola into a hyperbola; but there may be any number of them at any point in the cone. In some popular astronomies a vertical section only is said to be a hyperbola, but that is a mistake. Its legs continually approach two straight lines called *asymptotes*, which are parallel to the outline of the cone, but they never reach them, because they approach with a continually decreasing curvature: just as a series of weights of a pound, half a pound, a quarter, and so on,

* Mathematically a hyberbola is double, having two foci, like an ellipse turned inside out, but the difference (instead of the sum) of the focal distances is constant: which, you will easily see, makes a pair of equal curves: but we only want one of them.

approaches 0, and their sum approaches two pounds, but neither limit is ever reached.

A parabola and hyperbola can theoretically be described by strings from the focus or foci, something after the manner of an ellipse, but not practically. The focus of all the four conic sections is also the point where they are touched by a sphere which touches the hollow cone all round, like a ball put into a wine-glass: if you draw the outline of the cone and the axis of any conic section, such a sphere is represented by a circle, which is easy enough to draw by trial.

The only thing I have to say about oblique cones is to reconcile what is said about the shape of countries being correctly represented in a stereographic map (p. 15) with the fact that every perspective view of a circle is an ellipse; and one perspective view of every ellipse is a circle. The explanation is, that an oblique cone allows two sets of circular sections, leaning opposite ways with respect to the axis of the cone; and in the stereographic projections of a small circle of the earth, not being a parallel of latitude, upon the plane of the equator, looking from one of the poles, the oblique cone from the eye to that circle cuts the equator, or any parallel plane, in a smaller circle turned the other way; and therefore the shapes of countries in such a map are truly represented, though some of them must be on a larger seale than others.

The particular conic section in which a comet, or a planet theoretically, may move depends on the velocity with which it happened to be started in some direction round the sun, and also on the direction. It will be shown at p. 303 that it could only go in a circular orbit

U 2

if it was started at right angles to the radius vector and with a velocity^2 = sun's mass ÷ distance. It can also be shown by mathematics that if it is projected at right angles to the sun with twice that velocity^2, i.e. with 1·414 times the velocity due to a circle, it will go in a parabola of which the place of projection is the perihelion. We may observe too that that is the velocity with which a comet or a meteor would reach that point if it came from an infinite distance straight at the sun as at p. 111. If the velocity at perihelion is anything between those two it will describe an ellipse, which may be nearly either a parabola or a circle; and if we found a comet travelling anywhere with a velocity^2 greater than twice the sun's mass ÷ the distance we should know at once that it is performing a hyperbolic orbit. In either that or a parabolic one of course the comet never comes near the sun again. We cannot go into cases of oblique projection without mathematics. In all these calculations the sun's mass has to be reckoned in terms of its attractive force at the unit of distance, as we shall see in the next chapter.

COMETS.

Less is known of the nature of comets, and especially their tails, than of any bodies in the solar system and even some beyond it. Their general character is that they have what is called a nucleus or bright centre, but transparent, so that stars are seen through it, and their rays are not refracted in passing through it eccentrically: then of a less luminous *coma*, or head of hair, round the nucleus; and then, most large

comets at a certain period of their approach towards perihelion throw out a long tail, which swings round the sun with them, nearly always pointing from the sun, but generally curved a little as if the end lagged behind in swinging round. A very few comets have had tails pointing towards the sun. The matter of the tail generally appears to be first thrown out of the comet on the side next the sun and then to be violently repelled to the back side by some repellent force in the sun. But no one knows why that should be so, while the whole mass approaches the sun under the ordinary law of attraction just as if it were a planet in a very eccentric orbit. For the orbits of all the great comets have been very eccentric, and some parabolic, and even hyperbolic; i. e. they can never visit the sun more than once in their lives, but afterwards run off until they perhaps come within the range of some stronger attraction from another sun or star: otherwise they might go to an infinite distance.

This repulsive force of the sun must be something quite different from an explosive force, like that of our volcanoes, which is temporary, and only prevails over gravity for a short time. It is proved by what we saw of some of the envelopes of the sun that there is a repulsive force there; and a temporary or explosive force would be sufficient to produce those effects, but not the effect of comets' tails. There are various other difficulties about them. It is inconsistent with the laws of gravity that a tail consisting of any kind of matter retaining its personal identity (i. e. being the same matter day after day) can swing round the sun;

for they are sometimes one or two hundred million miles long, and the farther end ought by the law of gravity to go much slower than the head, instead of faster, as the apparent far end of the tail does; for after perihelion it is in advance of the comet and has gone through a much larger arc.

Therefore the idea prevails that the tail seen after perihelion, or indeed after any day, is not the same matter as that seen the day before; but is fresh, like the smoke or steam out of a locomotive chimney: so that you may say a comet is continually shedding the end of its tail by dissipation and renewing it by emission. Some tails are short and thick like a 'batswing' burner gas flame, and some are spread out in several distinct rays like a fan. 'Newton's comet' of 1780, the finest ever known, shot out a tail 60 million miles long in two days and twice as long afterwards. It went nearer to the sun than the moon is to us, and was exposed to a heat 25,600 times greater than the hottest sunshine in the tropics. A smaller one in 1843 went still nearer, within only 65,000 miles, and was exposed to a heat which would melt any substances we know of.

There was also a very fine comet in 1811; but the grandest of modern times is called Donati's, which many of us saw in the autumn of 1858 and nobody will see again for 2100 years, as it has to go 80 times as far as Neptune. Its tail was 30 million miles long and 90,000 wide near the head, and much wider at the thick part of the brush. And at one time it threw out two minor tails more rapidly than the main one. Some comets have a head 50,000 miles wide.

Sir J. Herschel says that about 36 periodical or returning comets are known, of which 5 have periods from 70 to 80 years and the rest from 3 to 7; and further, that there are only two of the whole 36, both long ones, that do not go round the sun the same way as the planets; and none of their orbits are very far from the ecliptic. This is now the more remarkable, because several comets are identified with meteoric orbits, which are innumerable and inclined at all degrees to the ecliptic. The whole orbit of a comet can be laid down from only two observations near perihelion. For they give its least distance and its velocity there, and that gives the eccentricity, or defines the parabola if the orbit is one; and the plane of the orbit must go through the sun's centre and the two observed places, and that gives its inclination to the ecliptic and its nodes. And by a more complicated process the orbit can be determined from three observations anywhere. I have seen no account of any particular comets with hyperbolic orbits. No other orbit than one of the conic sections is possible.

They are liable to fall under the dominion of any planets which they approach. The most famous case of that kind was Lexell's comet of 1770, which was found to be travelling in an orbit which must have had Jupiter at its aphelion, and had never been seen in our regions before, though it was too large to have been missed, especially with such a short period or such frequent returns. In 6 years it came again, but not the next time it was due: nor ever since. Its orbit had been made such by that first approach to Jupiter that they would concur again in one of Jupiter's

periods and in two of the comet's; for it went much nearer the sun than Jupiter does, and therefore had a much shorter axis major of orbit and period. Nobody knows what happened to it the second time of meeting Jupiter, nearer than one of his own satellites. I have already mentioned that the mass of Mercury was determined by its disturbances of comets. Their mass is far too insignificant to affect the planets in return, or even the satellites of Jupiter. You may see accounts of the aspect and behaviour of some other comets in Herschel's 'Familiar Lectures' and in several of Mr. Proctor's books, and in Lardner's 'Astronomy' by Dunkin, with pictures of some of the most remarkable.

The most singular performance of any comet was the separation of Biela's comet of 1845 into two, which returned in 1852, farther apart, but have never been seen again, though it was expected to come very close to the earth, if not to strike it: which however would be much less alarming now than when the tenuity of comets was unknown. Halley's comet, which followed Newton's in 2 years, with a period of 76, subject to delays by Jupiter and Saturn, in 1835 swelled to 74 times its perihelion size in 17 days after perihelion, and then got too thin to be seen any more. Other comets apparently contract towards perihelion and expand again; from which it is inferred that they are really expanded by heat there so much that their outer parts become invisible, and are gathered up again afterwards by attraction.

As to their constitution, they seem likely to consist of the same elements as meteors, from the identification of their orbits; but the elements may be in a gaseous

instead of a solid state. It appears to be proved both by the spectroscope and by the tails emitting polarized light, that the tails shine by sun-light reflected, while the head is self-luminous, as an inflamed gas. But the outside of the coma sometimes gives a complete coloured spectrum, as if it consisted of solid particles incandescent, among which carbon is conspicuous by its distinctive 'lines.' Their perihelion expansion also tends to prove them gaseous, but not their transparency to stars, as that might come from the particles being too widely spread to prevent rays coming through the whole mass, though it may be 50,000 miles across; and in that case star rays passing through eccentrically would not be refracted (and they are not) as they would in passing through a globe of gas. On the other hand, no kind of force is known which would keep solid particles far apart against their mutual attraction, as the natural expansibility of gases does.

CHAPTER V.

THE LAWS OF PLANETARY MOTION.

NEWTON propounded three fundamental laws of motion at the beginning of his Principia. The first has been already noticed at p. 29, that a body will go on moving for ever in a straight line under any impulse with uniform velocity until some new force comes to alter its direction or velocity. The second is that any new or second force draws the body aside from a straight line just as much in a second of time as if the body had not been moving; so that at the end of the time it is just where it would have been if the first impulse had sent it straight on, and then the second force had drawn it aside from there; or conversely, as if the second force had first pulled it in *its* direction in no time at all and then the first had sent it forward in a second. And that comes to the same thing as if the body had moved along the diagonal of a small parallelogram of which one side is the course it would have taken under one force and the adjacent side the course it would have taken under the other force.

I said a *small* parallelogram, because curvilinear shapes and motions are dealt with in mathematics by supposing them cut up into such little bits or 'ele-

ments' that each may be treated as a straight line or bounded by straight lines. That is the principle of the 'differential calculus,' and the putting together of such little bits (a much more difficult process) is the business of the 'integral calculus.' These also were invented by Newton, though in a somewhat different form from what is now used. It follows from the second law that any force or motion in an unit of time may be split up or 'resolved' into two, forming the adjacent sides of a small parallelogram of which that is the diagonal, both in magnitude and direction. And in fact this, which is called the diagonal of forces, is the practical form of the second law of motion. The third, that action and reaction are equal, is self-evident, and of very little use; and some mathematicians have tried to substitute another for it, but we can do very well with two.

The truth of the second law is sometimes illustrated in lectures by a machine which is contrived to shoot one ball forward horizontally at the same moment that it drops another, and you hear them both fall together, though the shot has gone many times farther than the other ball. Taking the motion of the earth and sun together, the earth at one time of the year is rushing through space at something near 2000 miles a minute, and at another time very little, and yet things fall to the earth in the same time always. There have also been mathematical proofs invented that this law of motion must be true. Newton propounded it as an axiom, which means a self-evident truth incapable of deduction from any other.

Reverting to what I had to say of the law of gravity, at p. 24, it is convenient to add now, that the term

momentum, which you often see, means the mass × its velocity, which is also called its 'moving force' and its 'quantity of motion.' But it is not that which represents the work done in lifting it, or which it will do in falling again. That is represented by mass × ½ velocity², and is called vis viva, and sometimes its 'force of motion;' and of late, 'kinetic energy;' for velocity may be proved to = gravity × twice the height fallen through. This means the final velocity when stopped; and 'gravity' means the constant force of it near the earth's surface. When the force varies the result is different, except for a very short time. Again the vis viva of rotation, as of a fly-wheel or of the earth, is half the sum of each particle × its own linear velocity², or × its distance² from the axis of rotation × velocity² of rotation: which therefore depends on the shape of the body. Also the sum of the mass of each particle × its distance² from the axis constitutes its *moment of inertia*, or its resistance to being set in rotation, or stopped when it is rotating. This is evidently greater in an oblate spheroid than in a sphere of the same weight; and in either case it is two thirds of what it would be if the whole mass were condensed into a thin band round its equator.

Consequently the moment of inertia of a shrinking globe keeps decreasing, and the time of its rotation would decrease also (p. 285); for each particle would still try to move through the same space in a second, and when the radius is diminished a given length of arc corresponds to a larger angle or angular velocity. So if you swing a small weight by a long string round your finger, it revolves faster as the length of the string

diminishes by winding up. And therefore in considering the earth's retardation by the tides, in connection with the moon's secular acceleration, we must bear in mind the possibility of the earth still shrinking a little, which would tend to accelerate it; and on the other hand the possibility of its enlargement by the continual addition of meteors, which must tend to retard the rotation by increasing the mass of the earth. But both these effects are probably extremely small, if they exist at all.

Centrifugal force is another thing of which it is necessary to have a clear notion. Strictly speaking there is no such thing as any force or tendency in a revolving body to fly away from the centre. The only tendency is to obey the first law of motion and go on in a straight line. If a string breaks by which you are swinging a weight round, the weight goes on in the direction of a tangent to the circle at the place where it breaks loose, and the art of slinging depends on knowing where to let it loose, so as to send the stone forward in the right direction. It may be defined as being equal and opposite to the force required to draw the body out of the straight course into the curve in which it is constrained to go. The force which counteracts the so-called centrifugal force of the planets is the sun's attraction, which may be considered a kind of elastic string, allowing some variations of distance or *radius vector*, but always bringing them back again.

Centrifugal force in a circle is easily calculated, and we can only deal with the orbits of the planets and their moons as circles, without far more mathematics

than can be introduced here. All force is measured by the velocity which it imparts in some small unit of time such as a second, during which it may be considered constant, whether it is so for a longer time or not. Since the velocity has to increase from 0 up to what it is at the end of the second, the body would have moved twice as far as it does if it had been going with that velocity from the beginning. Therefore the velocity which measures the force is twice the space actually moved through in the first second. Centrifugal force then is twice the distance of the end of the arc moved through in a second, from the tangent or straight line in which the body would have gone if it had not been drawn aside into the curve. You must accept it as easily proved by geometry that twice that distance = arc^2 ÷ radius; or centrifugal force = velocity^2 in the curve ÷ the radius of curvature at that point, i.e. the radius of a circle which has the same curvature as the orbit there. And since linear velocity (in a circle) = radius × angular velocity, centrifugal force also = angular velocity^2 × radius of curvature: which is a different thing from the radius vector or distance from the centre of force, and does not even coincide with it in direction, except at the apses of the ellipse, and even there does not coincide in length.

In a non-circular orbit angular velocity bears no such simple relation to linear; but this rule always holds: linear velocity varies inversely as a perpendicular from the centre of force drawn to a tangent of the curve at the place where the body is. In an ellipse (see p. 305) the tangent TP*t* is easily drawn from the fact that it makes equal angles with the two focal

distances SP, HP; and ST the perpendicular always meets it in the circle which contains the ellipse.

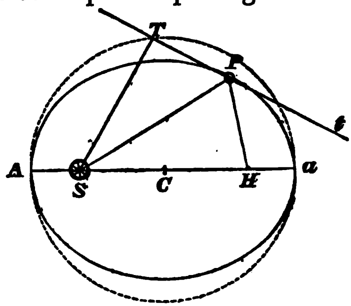
Circular orbit: Law of Time and Distance.—A planet cannot perform a circular orbit unless it has been 'projected' at some time or other in a direction at right angles to its sun's distance and with just that velocity which would give it so much centrifugal force as requires exactly the sun's attraction to deflect into a circle from the straight line. We saw that centrifugal force in any circle is velocity^2 $\div$ radius. And the sun's attraction is his mass $\div$ distance^2, i. e. radius^2 of that orbit. Therefore linear velocity^2 must = sun's mass $\div$ distance, for the orbit to be a circle; and therefore angular velocity^2 = sun $\div$ his distance^3. Angular velocity is the angle, or fraction of 360° or 2π or 6·2832, moved through in a second; and therefore the period (in seconds) $= 4\pi^2$ (or 39·48) $\times$ distance^3 $\div$ sun's mass: which is the law of time and distance we have often referred to, and have now proved—at least for circular orbits; and it is demonstrable (but not without mathematics) that it holds equally for elliptic ones. We shall see afterwards how the sun's mass is to be expressed for this purpose in feet or miles per second; for it is evident that it will not help us to say that it is so many times the weight of the earth, or so many tons, but we must measure its attractive force by something reducible to feet per second.

Elliptic orbits.—But if a planet was at any time projected, either not exactly at right angles to the line from the sun, or with any other velocity than that just now ascertained, it will evidently perform some other kind of orbit. If the velocity of projection at perihelion

was twice as much as that required for a circle the planet would leave the sun for ever, as I said at p. 292, but anything between that and the circular velocity will make it revolve in an ellipse more or less elongated, of which the same place will always be the perihelion, subject to gradual variation by disturbances of other planets. But this cannot be shown without mathematics of rather a high order, or else by long and complicated geometrical contrivances, which Newton invented for public demonstration, though he privately invented others which ripened into the differential and integral calculus. All that we can do here is to show how planets are brought back towards the sun after running away from him towards aphelion, and again how they escape falling into the sun after approaching him with continually increasing velocity for half their year, making an orbit which we otherwise know to be elliptical with the sun in one of the foci.

We saw at p. 298 that as two forces can be compounded into one, which is called their *resultant*, so one force can be divided or resolved into two in any directions we please; except that there can be no resolved part of any force in a direction at right angles to its action. That is the reason why we need not consider the resolution of forces in circular motion; for the motion in a circle is always at right angles to the radius or to the direction of the central force. But in an ellipse it is not so, except at the two apses. If you look at this figure of an ellipse, you see the motion of the planet P in either direction, towards T or *t* (both in the tangent to the ellipse), is not perpendicular to PS, the radius vector. So that if it is going in the direction PT

part of the sun's force goes to increase the velocity in that direction, while the other part is pulling it out of that direction. The sun's force then may be resolved into two forces, one in the tangential direction PT, and the other in the direction TS perpendicular to PT; and these two forces are proportionate to those two lines. If the planet is going away from the sun towards *t*, the sun's force is resolved in the same way, and is still represented by TS and PT, only the tangential part of it retards instead of accelerating.



Then while it approaches the sun S, or while the radius vector SP diminishes, the force represented by PT continually augments the velocity, till at last at some point A the velocity becomes so great that the centrifugal force first balances and then beats the centripetal; and so that point becomes an apse, or a place where the radius vector changes from decreasing to increasing, or vice versâ. So we may say that the sun enables the planets to run past him at perihelion by having made them run faster towards perihelion—only not *straight* towards him. The converse of this takes place when the planet is approaching aphelion, in a direction like P*a* (but on the lower side of the figure, or reversed). Then the sun has been gradually reducing the velocity by that part of his force which is resolved into the direction PT, now acting against the

increase of the radius vector. At last the velocity is reduced so much that attraction prevails over centrifugal force, and the radius vector can increase no longer, but begins to diminish again at *a*, the farther apse, and then the planet begins to approach the sun again, and so the orbit is completed. But why the apses recur at the same places, under a force varying exactly as the inverse square of the distance, and why the orbit is an ellipse and not some irregular or egg-shaped oval, cannot be explained without mathematics; and even they will only prove that it is so.

I showed in the moon's disturbances (p. 197), that if the central force is less at aphelion than it would be according to the square of the distance, that is, if it varies in a rather higher ratio, the apses do not recur at the same places, but advance, and the orbit is a kind of spiral returning across itself, that is, a revolving ellipse. I now add that if the central force varied in as high a ratio as the inverse cube of the distance (as disturbing forces do), the centrifugal force and the force of attraction could never balance each other again if either of them once got the best of it; and so the orbit would become a complete spiral, never returning into itself again, and the planets would either run at last into the sun, or else farther and farther off into infinity, according to which force had accidentally preponderated for a moment.

The only other law of force of which we have any experience, and that only as an indirect result of the universal law of gravitation in some special cases, is when the force varies directly as the distance, as in the case of a pendulum bob, which is attracted to the centre of

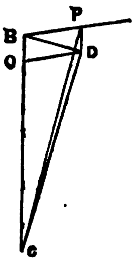
its vibration by a force very nearly in proportion to the distance from it, whether the pendulum is vibrating in one plane, as in a clock, or revolving as a conical pendulum; and in a mass of stars, as we shall see afterwards. It is singular that in that case also an ellipse is described, though the law of force is so extremely different; only the centre of force is then in the centre instead of the focus of the ellipse. Moreover attraction then = distance × some constant quantity (no matter what); and it must = centrifugal force, or distance × ang. vel.^{2}. Therefore ang. vel.^{2} = that constant quantity; or all the planets would have the same angular velocity and period under that law of attraction.

Kepler's Laws.—I said at p. 209 that Kepler discovered by observation that the planets' orbits are ellipses; and that is called one of Kepler's laws. The second was that the radius vector from the sun to the planet sweeps over equal areas in equal times, in the same orbit, but not in different orbits; which is called the conservation of areas; and it is the same thing in other words as saying that the angular velocity in any given orbit varies inversely as the square of the distance.* This looks as if it had some relation to the central force varying inversely as the square of the distance; but it has none, and would be equally true with a central force following any other law.

This cannot be proved without a little geometry, but it is so easy that I had better give it. Suppose that

* This is not inconsistent with the angular velocity^2 varying inversely as the cube of the distance in different orbits round the same sun (p. 303).

a planet would go in the direction BP (a very small arc performed in a second) if there were no sun at C. But there is; and suppose the sun's attraction would draw it through a space = BQ in a second; then by Newton's second law of motion (p. 299) if you draw PD parallel and equal to BQ, the planet will find itself at D at the end of a second, having travelled through BD. Then by a well known proposition in Euclid the area of the triangle BPC = that of BDC, because they lie between parallel lines, and have the same base BC; i. e. the central force has made no difference in the area swept over by the radius vector; for though it has shortened it it has made it wider. Again the area of BDC is half of BC × distance between BC and PQ, which distance is practically BC × the numerical value of the angle BCQ which represents the angular velocity in a second; and as that area is constant it follows that the angular velocity varies inversely as the BC^2 of every place, i. e. as the sun's distance.[2]



Kepler's third law, also discovered by long observation and trial of numerous conjectures, was that the square of the time or period of the planets varies as the cube of their distances. For this purpose you must compare any two or more of them. Thus, omitting fractions, the cube of Mercury's distance in millions of miles is $35^3 = 42,875$; and of Venus's $66^3 = 287,496$, which is about $6\frac{3}{4}$ times 42,875. Then Mercury's period^2 in days is $88^2 = 7744$, and Venus's $225^2 = 50,625$, which again is $6\frac{3}{4}$ times 7744. And you

would find the same rule held between any two or more of Jupiter's or Saturn's moons.

But this tells us nothing of the relation of any one planet's period to its distance and the sun's mass, which I have already proved at p. 303 for circular orbits. Newton also proved the necessary truth of Kepler's third law, and that the time of an elliptical orbit is the same as if it were the circle which contains it; or in other words, that the time depends only on the mean distance or semiaxis major, and not at all on the minor axis or the eccentricity. And therefore that law holds for all orbits round a centre of force varying inversely as the square of the distance. This proves what I said at p. 85, that the length of our year is a fixed and certain measure of the proportion of the weight of the sun to the cube of his distance, so that when one was reduced the other must be reduced also, in the same proportion.

WEIGHING OF THE SUN AND MOON.

I promised to prove that the sun is 322,700 times as heavy as the earth, and the earth 81½ times as heavy as the moon, and also to show how the sun's mass is to be expressed in the velocity per second which it imparts at any given distance. As a foot is the unit of distance generally used we shall have to express the sun's mass by the velocity it would impart to a body at that distance from the sun's centre if all his mass were condensed into a point there: which may be called the sun's absolute mass. We can also do the same for the earth and then compare them. But first

let us understand clearly what is meant by calling g, or gravity at the earth's surface 32·23 feet, considering the earth as a non-rotating sphere, so as to get rid of all the variations of gravity spoken of at p. 46.

That is proved in various ways. Stones dropped from a great height fall 16 feet in one second (omitting fractions), and 64 feet or 16×4 in 2 seconds, 16×9 in 3 seconds, and so on. That, by the same reasoning as at p. 300, mathematically proves that the force which makes them fall imparts a velocity of 32 feet per second in the first second, and adds that velocity in every following one. And this force g is necessarily twice the height fallen through $\div$ the time^2; because the velocity increases by arithmetical progression, and therefore varies as the square of the time (p. 112). And therefore you will easily see that the above figures (which depend on observation) make $g = 32$. Again it is demonstrable that gravity must $= \pi^2 \times$ length of a seconds pendulum, measured in the way which is explained in my treatise on clocks; and such a pendulum is found to be 3·26 feet in this latitude; and as π^2 is 9·86, that makes $g = 32·14$, and by proper calculation 32·23 if the earth were a non-rotating sphere. The meaning of gravity imparting a velocity of 32 feet in a second is that if gravity were cut off at the end of the first second a stone would fall only 32 feet in every following second. A machine called Attwood's, of which you may see a picture and description in the English Cyclopædia, was invented to show this experimentally, but rather for lecturing purposes, as the mathematical proofs derived from experiments on pendulums are much better.

We can now find the absolute mass of the earth in attractive force. Its radius as a sphere of equal weight would be 3958·6 miles; and gravity at the surface would be 32·23; and it = earth's mass $\div$ $(3958{\cdot}6 \times 5280)^2$ feet; which makes the mass = 14,076 billions of units of attraction, or so many feet per second.

The sun's mass may be found in several ways. The simplest and most direct is this. We found at p. 45, from the sun's mean distance and the length of the sidereal year, that the earth's mean velocity is 18·317 miles a second, and we know that the year would be the same if the orbit were quite instead of nearly circular. The centrifugal force in a circle is velocity^2 (or miles^2 per second) $\div$ radius or sun's distance $=$ (in feet) $\dfrac{(18{\cdot}317 \times 5280)^2}{92{,}000{,}000 \times 5280} = \dfrac{335{\cdot}52 \times 5280}{92{,}000{,}000}$; and this must balance or = the sun's attraction on the earth, or $\dfrac{\text{sun's mass}}{92{,}000{,}000^2 \times 5280^2}$; $\therefore$ sun's mass $= 335{\cdot}52 \times 5280^3 \times 92{,}000{,}000 = 4543{\cdot}6$ trillions; which is 322,700 times the 14,076 billions just now found for the earth.

Or thus: centrifugal force = distance $\times$ angular velocity^2, which must also = centripetal force or sun's attraction in a circular orbit = sun's mass $\div$ distance^2. Angular velocity per second is $2\pi \div 365{\cdot}256 \times 86{,}400$ (the seconds in a sidereal year); and $4\pi^2 = 39{\cdot}48$. Therefore sun $= \dfrac{92{,}000{,}000^3 \times 5280^3 \times 39{\cdot}48}{365{\cdot}256^2 \times 86{,}400^2}$; which you will find give the same result as the last process: in fact they are identical. Or again,

$$\frac{\text{Cent. force}}{\text{gravity}} = \frac{92{,}000{,}000 \times 39{\cdot}48 \times 5280}{32{\cdot}23 \times 365{\cdot}256^2 \times 86{,}400^2} = \frac{1}{1673{\cdot}7}.$$

But gravity=earth's mass÷its radius², and centrifugal force must = sun's mass ÷ his distance², and therefore we shall have $\frac{\text{sun}}{\text{earth}} = \frac{92{,}000{,}000^2}{3958{\cdot}6 \times 1673{\cdot}7} = 322{,}700$ as before.

This last calculation shows us also the proportion between the sun's attraction and the earth's at the earth's surface. For we see that if the earth were held fast, and not itself movable by the sun's attraction, gravity would be an 837th less at noon on the ecliptic than at midnight, because at noon the sun would be pulling things straight away from the earth with a force = the 1674th of the earth's attraction, and at midnight adding the same to it. But as the earth is itself attracted to the sun equally with the things on the earth, no such difference exists between the diurnal and nocturnal gravity.

Or again, if a pendulum could be freed from the earth's attraction, and subject only to the sun's, the force which makes it swing would be a 1674th of what it is; and as the time varies as $\sqrt{}$force, it would take 41 sec. instead of one to make one vibration. So small is the force which keeps the earth in its orbit; and that on Neptune is 900 times less. The tractive force on a fast railway train of 400 tons on the level is found to be about 4 tons; i. e. a spring balance between the engine and the train shows that tension. But if the earth were such a train it would exert a centrifugal strain of less than 5 cwt. on the rope which held it to the sun, though the earth goes 1000 times faster than the quickest train that ever ran.

The following is another way of weighing the sun,

but not so exactly; and it requires the moon's weight to be known first. The mean radius of the earth's orbit is 385·22 times that of the moon round the earth, and the moon's mean angular velocity is 13·37 times the earth's, being inversely as their sidereal periods. Therefore the moon's mean linear velocity round the earth reduced to rest (p. 24) is to the earth's round the sun, as 13·37 to 385·22. The sun's force on the earth is to the earth's force on the moon as their deflections from a straight line per second, which are as the squares of their velocities divided by the diameter of each orbit (just as the depression of the sea below the horizon = the distance^2 ÷ the earth's diameter, p. 5), and those diameters are as 385·22 to 1. Therefore the deflections are as $385{\cdot}22^2$ to $13{\cdot}37^2$. But the sun must be 385·22 times heavier than if he were at the distance of the moon, to produce the effect he does here. Therefore altogether $\dfrac{\text{sun}}{\text{earth} + \text{moon}} = \dfrac{385{\cdot}22^3}{13{\cdot}37^2} = 319{,}793$; and adding one 81·5th for the weight of the moon, that makes the sun = 323,716 times the earth alone.

You see this result exceeds the former one a little, though it is near enough for a general explanation. But we saw at p. 193 that the sun's disturbance lengthens the moon's period as much as if the earth's mass were reduced a 716th, assuming the distance to be unaltered. Therefore its real proportion to the sun is so much greater than it appears to be by calculation from the moon's actual period. If you reduce 323,716 by a 716th, the result is 323,168; which is again very near the former result; and we saw also that the exact

lengthening of the moon's period by the sun cannot be defined.

This same calculation evidently affords another mode of weighing the moon, if the sun has been first weighed independently; for the moon must be such a fraction of the earth's mass as will give the right result for the sun in the above calculation. In most popular astronomies the distinction between the earth and earth + moon is disregarded in weighing the sun; which makes a difference of nearly an 80th in his weight, besides misleading people as to the principle of measuring forces round a movable centre, which I have several times mentioned. Moreover the following method of weighing the moon without the sun is founded on that very distinction.

It is not the earth's centre E, but G, the centre of gravity of earth and moon, that really describes what we call the earth's orbit. Therefore the earth is before its mean place at first half moon, and behind it at second half moon, by the distance EG, which is now the thing we have to find, for we only knew it at p. 125 because we then assumed the moon's weight to be known. Now when Venus is at her nearest or 'inferior' conjunction, she is apparently displaced at half moons, first one way and then the other, by a parallax or angle which = EG divided by the distance of Venus from us; and as that is known, and twice that angle is quite large enough to measure, being nearly 1′, therefore EG can be found. Then we know that the moon is to

the earth as EG is to MG, and therefore the mass of the moon is found.

Weighing the moon independently.—But for the impossibility of defining the effects of the sun's disturbing force on the moon's period and distance, as explained at p. 193, the simplest of all ways of finding her mass would be the following. By the law of time and distance the absolute mass of earth + moon must $= 39{\cdot}48 \times$ cube of their mean distance in feet, divided by the square of the seconds in a sidereal lunation (2,360,591) diminished by a 716th for the increase of that period by the sun's disturbance. That makes

$$\text{earth} + \text{moon} = \frac{39{\cdot}48 \times 238{,}820^3 \times 5280^3}{2{,}357{,}310^2} = 14{,}240$$

billions very nearly. But we found the earth alone to be 14,076 billions, or 164 less than earth + moon. That difference, or the moon's mass, is therefore an 85th of the earth's mass; which is a tolerably near approach to the established proportion of an 81·5th or ·0123. Conversely, from that known weight of the moon, we may easily calculate that the 14,240 ought to be 14,249, and that her period is really increased a 630th by the sun. The advance of the apsides, or rather the causes which produce it (p. 197) must contribute something to that increase, making the orbit not quite an ellipse subject to the law of time and distance. We saw just now that the same difficulty prevents the sun's mass from being accurately determined from the moon's period and distance.

You may think however that we have been only reasoning in a circle, because we have reduced the moon's period to meet its increase by the sun's dis-

turbance, and yet that calculation of the disturbing forces at p. 192 assumed the moon's weight to be known. But there is nothing in that objection; for if you omit the moon's mass altogether in calculating the disturbances, putting 322,700 instead of 318,740 for the sun's mass, it makes a quite insignificant difference in this result.

The mass of Jupiter, or any planet with satellites, is found from the distance and period of any of them, by a calculation like that which I gave just now. For Jupiter's absolute mass, or force at the unit of distance, $= 39{\cdot}48 \times$ the cube of any of his moons' distance (in feet) divided by the square of its period (in seconds), which you will find in the table at the end. And then dividing the result by the 14,076 billions for the earth's mass you have the proportion of Jupiter to earth.

Or the mass of Jupiter may be compared with the sun, through any of his moons, as we compared the sun with the earth through our moon; except that Jupiter is too heavy to be neglected in comparison with the sun, and his moons may be neglected in comparison with him—just the converse of the earth. The result is,

$$\frac{\text{Sun}}{\text{Jupiter}} = \frac{♃ \text{ \& } \odot\text{'s distance}^3 \times ☾\text{'s period}^2}{♃ \text{ \& } ☾\text{'s distance}^3 \times ♃\text{'s period}^2} - 1;$$

the 1 meaning only that Jupiter's mass is not neglected. This ought to come 1270 or nearly so, and 308 for the proportion of Jupiter to the earth, as stated at p. 217. For Jupiter's mass has been also verified several times from his disturbances of some of the asteroids, and all the calculations give nearly that result.

The satellites of the greater planets are too small

compared with their primaries to be weighed through their own periods by the method given for our moon just now. The masses have to be found from their mutual disturbances. I have taken the latest results as given by Mr. Proctor in the R.A.S. 'Notices' of 1872. And so the planets without moons have to be weighed from their disturbances of each other, and of comets. All that belongs to very high mathematics; but I have now shown you how the dimensions and weights of all the solar system are measured ultimately by a foot rule and the vibrations of a pendulum.

Laws of Stability.—I will finish this account of the laws of motion of the solar system by stating three remarkable laws of its permanent stability or equilibrium; which however are found to be only approximately true, and more so for the large planets than the small (Challis's 'Principles,' p. 137). The first is, that if you multiply the mass of each planet (taking the earth, or any other, for the unit) by the square root of its mean distance from the sun, and by the square of the eccentricity of its orbit, the sum of all those products is invariable. The second is, that if you multiply each mass by the square root of its mean distance, and by the square of the numerical value (p. 9) of the inclination* of its orbit to a certain fixed plane near the ecliptic, the sum of those products also is constant.

The third is the most striking of all. The sum of all the areas swept over by the radius vector of every planet, satellite, and comet in the whole system, in any given time, is invariable, though they all may be made

* Strictly it is the trigonometrical *tangent* of the inclination, but the angles are too small for this distinction to be important.

to deviate a little from the law of the conservation of areas (p. 307) by their mutual disturbances. The area so described by any planet in a second, being constant in the same orbit, is the area of the whole orbit divided by the number of seconds in its year; and that area is 3·1416 × the product of the two semiaxes, or × the square of the mean distance if the orbit may be treated as a circle. From this, and the law that the square of the period varies as the cube of the distance, you may easily deduce another, that the areas described in different orbits in any time round the same central mass are as the square roots of the distances. Therefore the *areal velocity* of distant planets is greater than of near ones, though both their angular and linear velocity is less.

THE STARS AND NEBULÆ.

Hitherto we have been dealing with bodies whose motions are manifest and measurable, and their distances and sizes also measurable with considerable certainty. We have now to consider the infinitely larger number of luminaries of which all but very few are so distant that we can say nothing of their distance, except that it is certainly many million times the sun's; and even those few are so far off that only the finest observations can perceive any parallax or apparent change of place among them while the earth travels across a diameter of 184 million miles: which means that a sun filling up the whole of the earth's orbit would not appear to the nearest star as large as the planet Neptune does to us, which

is quite invisible without a good telescope. For no star has a parallax as great as 1″, the apparent radius of Neptune; and the parallax of the stars means the apparent size of the radius of the earth's orbit seen from them. You remember that parallax within the solar system means the apparent radius of the earth only. The one star with a parallax of nearly 1″ is called *a* Centauri, and the next nearest, 61 Cygni, is nearly twice as far; Procyon and Sirius* 4 times: then *a* Lyræ and 70 Ophiuchi are above 6 times as far. Or the sun would have to fill up more than the orbit of Jupiter to appear to those last stars as large as Neptune does to us. Arcturus and a few others have a parallax just visible, but too small to be measured with any certainty.

Now let us see what this parallax of 0″·976 for the nearest star means in actual distance. That angle, by the method explained at p. 9, has for its numerical value ·0000047; or the star's distance is 211,140 times the sun's, which makes it nearly 20 billions of miles; and the nearest star is 7000 times farther off than the farthest known planet.

And as light takes 8¼ minutes coming from the sun, it must take 3½ years to come from the nearest star, and 6½ years from the next; 14 from the next, and 20 years from the other three whose distances are just measurable, and still longer periods from all the others. As Mr. Froude says in his lecture on the *Science of History*, 'as the stars recede into distance

* The elements of Sirius have been altered by some further discoveries since the last edition. See Proctor's 'Our Place among Infinities,' p. 166.

time recedes with them; and there are stars from which Noah might be seen stepping into the ark, and Eve listening to the temptation of the serpent.' How far they do recede we cannot tell; for every increase of telescopic power only brings more stars into view.

There are indeed perfectly black places in the sky, in some instances called 'coal sacks,' with not even a nebulous haze, which may be 'star dust,' visible within them; but that does not prove that there are no stars there beyond the range of vision. For the light from each star may be spread so thin at this distance from it that the largest telescope-full gathered into a focus makes no impression on our eyes. Again, light may waste in coming, the undulations of the luminiferous æther being worn out by friction, like waves in a pond. Indeed if it does not, and if the stars are infinite in number, and if their distances apart does not increase with their distance from us—and there is no reason why it should—this curious result must follow, that the sky would always be as bright as if it were full of suns.

For in that case every line of sight must reach a star somewhere in the infinity of distance and the sky would appear quite full of stars. And though the light received from each star would be inversely as its distance^2 yet the number of stars at any distance would be directly as the distance^2, and every star would hide four behind it at twice its distance, and so on. Therefore the light would be the same from every patch of sky, whether the stars seen thus are far or near. And if they have generally the same intrinsic brightness as the sun, the effect would be the same as from a sky full of suns,

and those not round but hexagonal or square so as to fit close together. The fact however is so different from this, that we must conclude either that the stars are not infinite in number and extent, or else that light does waste in coming. We shall see indeed that they are neither equal in intrinsic brightness nor in distance, but the difference is by no means great enough to account for the visible condition of the heavens if the other two assumptions were correct.

In considering the question of a possible infinity of stars, i. e. of there being no distance in space where there are none, we must remember that that implies a corresponding increase of time for light to come, even if it does not waste in coming. And as the stars must have had a beginning, it may well be that there are multitudes beyond number, whose light has not yet reached the earth. Therefore all we can say is that there is no reason why the stars should not be infinite, or the whole of space (which must be infinite) occupied with stars as thickly as those we see. It is estimated roughly that about 5 millions can be seen with a good telescope far below the largest.

Until lately it was assumed, not as a fact, but provisionally as a basis for other reasoning, that all the stars would appear equal at equal distances; and it followed that if they were, their real distances would vary inversely as the square root of their apparent brightness, which can be measured by photometric contrivances, but with no great accuracy; and that is the meaning of the conventional word 'magnitude,' by which everybody knows that the different orders of stars are denoted. For instance, there are 5850 stars

of what are called the first six 'magnitudes;' of which only 24 are of the first, 60 of the second, 200 third, and so on, and about 15000 altogether above the 9th magnitude. Sir J. Herschel and other astronomers proposed various methods of classification or assignment of magnitude; which were interesting and important so long as the assumption prevailed that there was some relation between apparent magnitude and real distance. But Sir J. Herschel himself afterwards admitted that there was no foundation for that assumption, and that there was 'every reason for believing that the distance, the absolute magnitude, and the intrinsic brightness, may differ in different stars in the proportion of millions to one.' *

It should be understood that this word 'magnitude' has no relation to apparent telescopic size; for although we can now prove that some stars are much larger, or at least heavier than others, that is done by no direct observations of their size. The largest star seen in the best telescope is, or rather ought to be, nothing but a point 'of no parts and no magnitude', as Euclid says; and the better the telescope the smaller the star looks, not the larger. The apparent disc of a star in any telescope is nothing but an optical imperfection, and it is therefore called the 'spurious disc,' and is due to the fact that no telescopic glass yet invented brings all the rays of light from any point to an absolute focus in any other, as is done by the only perfect optical instrument, the eye. The largest star, measured by other tests than this, could have no sensible diameter in any telescope which gave a true image of it.

* 'Outlines of Astronomy,' p. 563 of all the later editions.

It is singular that no systematic attempt to exhibit and compare the stellar condition of various portions of the sky, beyond general statements about the Milky Way, appears to have been made till recently by Mr. Proctor in his Star Atlas, and the conclusions he has drawn therefrom.* The Herschelian theory of the Milky Way, which had hitherto been followed in all astronomical books, was that the appearance of that Galactic circle (as it was called in finer language) was due to the aggregation of an innumerable quantity of stars in a form something like a large thin grindstone, of which about half is split and opened out a little, or a 'cloven disc' as it was called, presenting a section like a tuning fork, or a man with his legs apart. Mr. Proctor however has shown, by reasoning which has never been impeached, that observation does not support that theory, beyond the fact that the Milky Way does consist of stars much more thickly strewn than they are on the rest of the heavens; but that the true shape is far more probably a kind of spiral, of which we cannot know the complete shape because we only see it edgeways. He gives a picture of one possible form of the spiral, though there are others which would do. I cannot give any fair abstract of his reasoning here, except at too great length, but it is such as requires no mathematical knowledge to understand it.

He also points out that the fact is, contrary to the general statements previously received, that just outside the borders of the Milky Way the stars are unusually thin, as if that mass had drained the neighbourhood by its attraction. Not only is the Milky

* See his 'Essays on Astronomy,' p. 328, or 'Other Worlds,' p. 258.

Way composed of innumerable multitudes of stars, both large and small, but it has twice as many bright stars as are due to its space, according to the average of the whole heavens. On the other hand, there are patches like black holes in it which contain no stars or scarcely any. His maps, already noticed at p. 16 for their principle of construction, also show that there is a large region round the south pole twice as rich as the average, and another smaller one in the north hemisphere about the same, while other parts of the celestial globe are very poor in stars, and those are closest to the rich parts, as in the case of the Milky Way.

Looking generally at these results, or at the sky itself, it may seem at first that there is nothing in them but what might be expected if the stars had been thrown there by chance like handfuls of seed upon a field. It would be sure not to fall uniformly, but thicker in some places than others. But Mr. Proctor calculates the chances of such aggregation and separation as exists among the 5850 bright stars, and he says that the fraction representing that chance would have 1 for the numerator and a denominator of no less than 132 figures; in other words, it is absolutely impossible that the distribution can be due to anything but some physical cause, though science is not yet far enough advanced to discover it.

He also shows that very exaggerated ideas had been formed by Herschel and adopted by others, of the necessary distance of the small or faint stars which compose the Milky Way. It is singular too that the parts of the sky which are poorest in stars are richest in nebulæ, which we shall consider presently; but I

mention that here in connection with this question of distribution according to some law or cause which has yet to be discovered.

Motion of the stars.—Though we call the stars fixed because we do not see them change their places, yet in fact they are all in motion. Indeed they must be; for if they had no motion of revolution round something, to balance their mutual attraction, they must all be rushing together with ever increasing velocity to some centre of universal destruction. The stability of the universe depends not on rest, which is impossible, but motion. It is true that we do not yet know round what centres the stars revolve, or what is the order of their motion generally. The theory that they all revolve round the star Alcyone is now refuted. It is certain too that they are not all moving towards any common centre, for the varying motions of many of them are ascertained, and indicated in Mr. Proctor's maps. Some large stars near together move in various and even opposite directions. Nevertheless so many in a group often have a common motion that they are evidently travelling together under the influence of some common force, though they are at immeasurable distances from each other and from us.

These are called the 'proper motions' of the stars, to distinguish them from the apparent motions, which arise from the fact that the sun with all his satellites is moving towards the constellation Hercules over about 150 million miles a year, or rather less than the width of the earth's orbit. Consequently the earth's real orbit in space is nothing like a circle or ellipse, but a twisted spiral, something like a distorted corkscrew

with the curves much steeper on one side than the other, since the sun's proper motion is very oblique to the ecliptic.

Until lately it was almost taken for granted that the small or fainter stars were always the more distant. But Mr. Proctor has made use of the investigation of their proper motions to refute that assumption also. The apparent motions of distant stars must evidently, on the average, be less than of near ones, especially as they go in all directions, so that no foreshortening due to the direction can account for the difference. He says he was surprised to find on the contrary that the average apparent proper motion of the small stars was equal to those of the first three magnitudes; and therefore, he concludes, 'there must be myriads of really small stars for every leading orb'* at all distances.

When a star's proper motion is directly to or from the earth, it is evident that it cannot be perceived. But perhaps the most wonderful feat performed with the aid of the spectroscope is actually measuring the direct approach or departure of a star. Dr. Huggins, P.R.A.S., has done this with several stars by a method of which I can only give an imperfect account, referring for a more complete one to his own work, or Professor Roscoe's book on the spectroscope, or to the article on Sirius in Proctor's Essays. The waves of light come rather faster from a source of light which is advancing than from a stationary or receding one, as the note of an approaching steam whistle or a bell is higher than from a receding one: which causes the pleasing variety of

* 'Other Worlds,' p. 267.

tone in a swinging bell. And though this difference is inappreciable in the colours of the spectrum, which are produced by waves of different velocities and lengths, it is not inappreciable in those lines called Frauenhofer's which we spoke of at p. 87. And by comparing the spectrum of Sirius with the spectrum of a fixed light in the same direction through the same prisms it is possible by great magnification to perceive that some well-known line is slightly shifted in the Sirian spectrum towards the red end, though the shifting is only the 250th of an inch. And it was found by proper calculation that that indicated a recession of Sirius from the sun of 29 miles a second, after making due allowance both for the earth's orbital motion at that time and for the sun's. If the bright line is shifted towards the blue end of the spectrum it indicates approach. Again combining that with the transverse proper motion of Sirius, of 16 miles a second measurable in the common way, it turns out that he has an absolute motion of 1000 million miles a year or 33 in a second, which is nearly twice the earth's velocity round the sun; and more than twice the sun's own motion, as just now described.

It is singular that this mode of measuring motion applies equally to stars whose transverse motion cannot be estimated because they have no measurable parallax, without which their apparent angular motion cannot be reduced to miles, except that we know it cannot be less than what is due to the smallest observable parallax of any star. Some other stars have been found to recede at various rates, up to about the same as Sirius, and others are approaching us still faster. In some cases stars which have always been considered optically of the

same family, such as Castor and Pollux and those of the Great Bear, are found to be moving opposite ways, both transversely and directly.

Double and coloured stars.—A great many stars, some say at least a quarter of them all, are not single, but consist of two, or three, and even more companions, so close together that it is one of the tests of a good telescope to be able to 'divide' such and such stars, though their distance apart really is enormous. And as many of these clearly revolve round each other, the connection is not merely optical, or arising from one being very nearly behind the other. About 600 brilliant double stars are known, besides the still more numerous faint ones, of which above 6000 have been counted in the northern hemisphere; and in 110 of these the companions have different colours, and one is nearly always brighter than the other, the fainter one being generally blue. And the brightness and colour often change as they revolve, both in double and in apparently single stars. Some appear to change their colour permanently, if old accounts can be relied on.

There is one remarkable case of an apparently double star of which the companions were once so far apart that they were considered independent, and called γ_1 and γ_2 Virginis, which the late Admiral Smyth, P.R.A.S., described in a letter to Mrs. Somerville in 1836 as having then lately come so close together that one totally eclipsed the other. The elliptical orbits of 16 or 17 pairs have been determined; but unless they are near enough to us to have a measurable parallax, we can tell nothing of their absolute distances apart, or (therefore) of their masses, as we shall presently for a few of them.

Everybody has heard of the 'lost Pleiad,' by which those once famous 'seven stars' have been reduced to six. And sundry others have gone out and come in within the time of history. In May 1866 a star in Corona Borealis blazed up from the ninth magnitude to the second, and then faded again, as others had done before. Examination by the spectroscope showed that this was due to some great outbreak of inflamed hydrogen. Nobody can tell that the same thing may not some day happen to the sun; and if it does, the inhabitants of the planets will fare badly. The star called η Argus varies from the sixth magnitude to the first. Whewell remarked that the periodical obscuration of Algol cannot be explained by the passage of a dark satellite over it, because such a satellite would have to be as wide as its own orbit to produce the observed effect; and therefore Algol must have one side darker than the other. Blue stars generally have a stronger red companion.

Many stars examined by the spectroscope are found to contain common elements of the earth, such as iron, sodium (the base of common salt), mercury, antimony, magnesium, &c., besides others not known here, because the spectra have lines corresponding to no known substances. But the absence of any line is not considered to prove the absence of the corresponding substance, as it may be there, but not in a state of incandescent vapour. Still, if there are elements in the stars which are not here, there may be well some here which are not in the stars; and it is said that no two stars have quite the same spectrum, so far as they have been examined yet.

Masses and brightness of the stars.—Now that we know Jupiter, and probably the other large planets, to be partially self-luminous, the question whether the smaller of a pair of double stars is to be called a satellite or a companion is little more than one of words. But wherever the star has a measurable parallax, and the nutation of either of them round their common c. g. can be also measured, it is easy to determine their masses, both joint and several, as soon as their period is ascertained. Calling the larger Sirius (1) and the smaller one (2), their distance apart is measured as 47 times our distance from the sun (which we take for the unit of distance now), and their period round each other 49 years. It follows by the same sort of calculation as at p. 303 that $\frac{\text{all Sirius}}{\text{Sun}} = \frac{47^3}{49^2} = 43{\cdot}25$. But Sirius_1 oscillates on each side of its mean place or common c. g. $16\frac{1}{4}$; and therefore the joint mass $43{\cdot}25$ has to be divided in the proportion of nearly 31 to 16; which makes $\text{Sirius}_1 = 28$ suns, and $\text{Sirius}_2 = 15\frac{1}{4}$.

But Sirius is estimated to excel the sun in light (of course at equal distances) no less than 192 times, and some observers say much more. If the intrinsic brightness of any given area of Sun and Sirius is equal (but there is not the least reason why it should be), that would make Sirius, not 28, but 2688 times as large as the sun, or its diameter nearly 14 times as great. But if so, its density can be little more than a 100th of the sun's, or a 400th of the earth's, or between a 70th and an 80th of the specific gravity of water. There is no known substance of that kind, being from 10 to 12 times as heavy as air, of which the

specific gravity is about an 840th of water. Therefore, if the observations are right on which those calculations are founded, Sirius must be immensely brighter intrinsically, or else his visible size or photosphere must exceed his solid size far more than the sun's does; and how much that is we do not know. He is now thought to have four more satellites, the smallest of which is bigger than the sun.

There seems reason to believe that Procyon and his satellite are still larger, and $Procyon_1$ nearly three times heavier than $Sirius_1$, though not so bright.* Astronomers have divided the joint mass between the pair in the proportion of 80 and 7; so that Procyon exceeds the sun about as much as the earth exceeds the moon. The following is a table of the few stars that have been weighed in this way; for though the periods and apparent orbits of many more are known, they have no measurable parallax, and therefore we know nothing of their real distances apart, or their masses. I have seen no division of any except Procyon and Sirius. The sun is taken as the unit of mass, and his distance from us as the unit of distance apart of the companions.

Stars.	Mass.	Period. Years.	Distance apart.	Parallax.
Procyon . . .	80+7	40	52	0·24″
Sirius . . .	28+15	49	47	0·23
70 Ophiuchi . .	3·1	96	30·5	0·16
α Centauri . . .	·41	81	15	0·98
61 Cygni . . .	·31	520	45	0·54

You see *α* Centauri and its satellite are not very different in period and distance from the Sun and Uranus. That star is said however to emit three times as

* R. A. S. 'Notices,' xxxiv., p. 26.

much light as the sun, though its mass is not much more than a third of the sun's. The light of 61 Cygni is said to bear about the same proportion to the sun's as their masses.

Of course the stars emit heat also. Mr. Stone came to the conclusion, from experiments, that we receive as much heat from Arcturus and from Vega as we should from a 3-inch cube of boiling water 383 yards off; but that again, Mr. Proctor observes, is far beyond the proportion due to their light, compared with the sun's, and all such estimates both of light and heat from very distant sources can only be regarded as rather wide approximations; see pp. 56 and 319 on this point.

NEBULÆ.

I have already spoken of the grandest of all the nebulæ, the Milky Way. Generally a nebula may be described as a bright patch in the sky, in which one or more stars may or may not be visible, and which may or may not be resolvable into stars or 'star dust' by telescopes of very high power. The success of Lord Rosse's great telescope in resolving some nebulæ which had defied all other telescopes led a good many people to jump to the conclusion that all nebulæ would be resolvable if we had only telescopes powerful enough to do it; just as a distant flight of birds, or a plague of locusts in the East, which looks like a black cloud, can be distinguished either by coming nearer or by being seen through a telescope, which magnifies the spaces between them.

But the spectroscope has settled that question also,

and has proved that many nebulæ are merely gaseous, because their spectra consist only of one or more bright lines; and some of them are ascertained to consist chiefly of nitrogen. But other nebulæ have a proper rainbow spectrum like any solid or fluid body heated, and some have an opaque nucleus within a gaseous envelope. They are also of various forms to which some odd names have been given, such as the Dumb Bell and the Crab. Some are round and are called planetary nebulæ, which seems to me a peculiarly inappropriate name, since they *wander* no more than the others, and roundness is no peculiarity of planets, and they are self-luminous, which planets generally are not, and they are not solid, which all planets are. Their smallness is only the effect of distance; for those which have an apparent diameter as small as Neptune (2″) have no discoverable parallax. If they had as much parallax as Arcturus, the smallest of those above mentioned, that apparent diameter would imply a real one eight times the diameter of the earth's orbit, or that 'small' nebula would nearly fill up the whole orbit of Saturn. But in fact it is much larger, and we cannot say how much, because it has no parallax at all.

Some nebulæ have spiral shapes like wisps or a mop in a state of rotation. No rotation has yet been observed as a fact; but the velocity of rotation of any system varies inversely as the square root of its moment of inertia (p. 300), and therefore of its density when the mass is given,* and therefore a very light nebulous

* This is only another form of the law of time and distance so often referred to, and proved at p. 303.

mass would have a very slow rotation; and as it also depends on the distance (distant planets travelling slower than near ones) a very large nebulous mass would revolve slowly for that reason also. Moreover the rotation of such a mass would have that mop-whirling appearance of the outer particles being left behind by the inner ones, for the inner ones must go faster to preserve the equilibrium between centrifugal force and attraction. At least it would be so if the density of the nebula increases greatly towards its centre, as in those whirling nebulæ it does, according to the pictures of them.

But we must now notice a peculiar result of the law of gravity on a mass of loose particles kept apart either by mutual repulsion like a gas, or by centrifugal force if they are all revolving. In a spherical mass of uniform density the attraction everywhere towards the centre varies simply as the distance from the centre, because the attraction of all the sphere outside each particle is nothing (p. 35). Moreover the attraction towards the axis of rotation in the plane of a parallel of latitude varies as the distance from that axis, either of a sphere or a spheroid. But centrifugal force also varies as the distance from the axis (p. 302) under any given velocity of rotation. Therefore a whole sphere or spheroid, however large, of equidistant particles or stars may rotate together without any change of relative position among the particles, and consequently without any motion of any of them being visible from the rest. But rotation would not prevent them all from gradually collapsing into a flat spheroidal mass in its equator under the mutual attraction of the particles in that

direction. No doubt however the stars are all moving under some law which is duly calculated to make them keep their distance and avoid temporary collisions and ultimate collapse. If the particles of such a sphere have also a repulsive or expansive force like a gas, acting against gravity, the problem is materially altered and depends on the law of expansion.

If the law is invariable that the expansiveness (independent of attraction) varies inversely as the space within which the gas is confined, doubling the radius of the sphere would increase the space eight times and therefore diminish the expansive force eight times, while the attraction to the centre (the mass being the same) would only be reduced four times, i. e. inversely as the square of the radius. Therefore there is some size of sphere at which the two forces balance each other, and a sphere of gas may have a definite boundary, as Professor Challis * somewhere says the earth's atmosphere must have—unless the law of expansion changes, so as not to decrease as the cube of the radius increases, while attraction decreases only as the square.

At any rate there is the fact that round nebulæ of gas are seen, shining by their own light. At one time it was thought that these might be masses of stars at some distance enormous even compared with that of individual stars. But Dr. Whewell in the 'Plurality of Worlds' advanced an argument fatal to that theory, even without the aid of the spectroscope, and applied it to two remarkable masses of mixed stars and nebulæ,

* I mention this because the fundamental idea of that ingenious book, the 'Fuel of the Sun,' was that the atmosphere is boundless, only decreasing in density with the distance.

both resolvable and irresolvable, which are called the 'Magellanic clouds,' in the southern hemisphere. The larger of them has an apparent diameter of 6°, or 12 times that of the moon, which means that it is a little wider than a tenth of its distance from us. That both the 'clouds,' and all the other round nebulæ too, should have the shape of long cylinders with their axes in the line of vision from here, is an improbability which amounts to impossibility. Therefore they must be spherical or nearly so; and in that case the depth of the greater Magellanic cloud (for example) is no more than a tenth of its distance. But it contains bright stars as well as irresolvable nebulæ; and an irresolvable nebula of stars must be, not one tenth farther off than the visible stars, but many thousand times as far. Therefore those nebulæ cannot consist of stars, if the visible stars there are really *in* the 'cloud,' and not merely in front of it, and so within it optically.

THE CELESTIAL GLOBE.

The thing called a celestial globe is a picture on a globe of all the stars as they appear in the sky. Properly of course such a globe should be hollow and we in the inside of it. And in fact all the pictures are reversed from what they appear in the sky. Take the well-known figure of the Great Bear or Charles's Wain, which is much more like the outline of a saucepan with a bent handle. As we see it in the sky, and in star maps, this handle, or the bear's tail of three stars, is on the left hand (except about midnight in winter, when it is turned the wrong way up), and the two stars

which form the outer edge of the saucepan, and are the pointers to the north pole star, are on the right hand. But it is drawn the opposite way on the celestial globe; which is covered all over with pictures of those Bears, Lions, Virgins, Scorpions and other things, into which the ancients—and some moderns too—fancifully divided the groups of stars and called them constellations. The particular stars are usually denoted by Greek letters attached to the Latin name of the constellation, as *α* (Alpha) Lyræ (the brightest northern star), *α* Centauri (the nearest), *β* (Beta) Leonis, *γ* (Gamma) Draconis, which was the 'pole star' 4000 years ago (p. 71), and some by their numbers in a constellation, as 61 Cygni, &c. A few have names of their own, as Sirius, Alcyone, Polaris, now the pole star within 1° 24′, but once 12° off the pole in consequence of precession; and it will be as near as 26′ in 360 years.

Another thing which a celestial globe has on it is *the Zodiac*, which is a band of 8° on each side of the ecliptic, or 16° altogether, and on which are the constellations of the 12 Signs of the Zodiac, and within which all the planets have their orbits, except some of the Asteroids. But as I explained at p. 66, the signs have left their constellations nearly 30° within the time of astronomical records. Therefore you see on the celestial globe the two crossings of the equator and ecliptic, ♈ and ♎, lying towards the left hand of the picture of the fishes and of the lady called Virgo, the Astræa of the poets, who holds the scales of justice (Libra) in her hand. Celestial globes, like terrestrial, are made to turn upon the poles of the equator, not on

the poles of the ecliptic. One would be just as right as the other for a celestial globe, only they represent different things: turning the globe on the poles of the ecliptic would represent the motion of the moon round us in 29½ days (subject to her slight departure of 5° from the ecliptic) and the apparent motion of the sun round us in a year.

Turning it on the poles of the equator represents the daily rotation, and enables us to see how the stars rise and set on any day, and the time they are visible. For though the sidereal time of rising and setting of the stars at any place is constant, the solar time of it varies with the sun; and the attitude of the earth with respect to the stars at midnight in summer is the same as at midday in winter; so that if the sun were darkened at noon any day the aspect of the heavens would be the same as it is at midnight six months off, except as to the moon and planets. For you may consider the earth fixed, and the sun going round it. If you want to find the aspect of the heavens at any given time here, elevate the north pole 51½° * above the northern part of the horizon, or the equator 51½° above the southern part. Find the place of the sun on the ecliptic from his longitude given in almanacs, and bring him down to the under side of the meridian and set the hour circle to XII there. Then the upper half of the globe shows the stars above the horizon at midnight; and by turning the globe with the hour circle, you will find those visible at any other time of night.

* This does not necessarily mean 51½ as marked on the brazen meridian; for they are not all graduated according to latitude, but some from the poles; or partly one and partly the other.

No stars within $51\frac{1}{2}°$ of the pole ever set here. They are only put out by the sun at different times daily; nor do any within $51\frac{1}{2}°$ of the south pole ever rise here by either day or night. At the north pole in winter they see all the northern hemisphere of stars, and no more, all through their six months' night. At the equator every place sees all the stars of both hemispheres every night, except those which they lose in twilight; for their horizon, formed by two opposite meridians, sweeps round the whole sky in their constant night of twelve hours; or half the stars rise and half set every night. Between these extremes there are all the variations due to latitude and to the sun's position north or south of the equator.

Right ascension and declination.—I have already said at p. 65 that longitude of the heavenly bodies is measured from ♈ on the ecliptic, and not on the equator from the meridian of Greenwich: which indeed would be nonsense, as Greenwich is changing its position with reference to every star every minute. Perhaps it might as well have been measured on the equator, as another name has had to be invented for the measuring of the stars along the equator from that point ♈ where all the celestial measures of that kind begin. That other name is *Right Ascension*, commonly written short R. A.; and there is no east and west in it: it runs on from 0 to 360° on the celestial globe, and from left to right, to correspond to the earth's moving as it does through space from right to left in this hemisphere (p. 41). The thing which corresponds to terrestrial latitude for the stars is called *declination;* that is their distance from the equator. R. A. is also

measured by time, like terrestrial longitude, at 15° to an hour.

There is also *heliocentric* longitude, which is the angle between two planes through the sun's centre and perpendicular to the ecliptic, one through the planet and the other through ♈; while *geocentric* or common celestial longitude is the angle between two such planes through the earth's centre. So the terrestrial longitude of any place is the angle between the two planes forming meridians, which both go through the earth's centre, intersecting all along the polar axis, one through the place in question and the other through Greenwich observatory, or whatever place each nation takes for its zero or 0° of longitude. The *declination great circles* for R. A. also intersect in the polar axis, only the zero is at ♈; but the *quasi-meridians* for celestial longitude intersect in the poles of the ecliptic.

Polar distance is the *complement* of declination, or the difference between the declination and 90°. This is so much used that it is abbreviated in almanacs for our hemisphere into N. P. D. So the *zenith distance* of a star is the complement of its *altitude* above the horizon. The distance of the pole, or pole star (if it were exactly there) above the horizon, or the distance of the zenith from the equator, is the latitude of the place where you are; and the distance of the pole from the zenith is the *colatitude*, or the difference of the latitude from 90°.

The 'horizon' in astronomy does not mean the accidental boundary of your sight by earth or sea, but a plane through the middle of the earth parallel to the 'level' surface of water or mercury where you are, or

to which the plumb-line is perpendicular (see p. 4), and extended to the sky. The *dip of the horizon* is the angular distance that you can see below the hemisphere of sky of which the base is the level plane through your eye; and it is evidently greater the higher you are above the earth. The lowest point, opposite to the zenith, is called *nadir*; both of which words came from Arabia, like the whole contrivance of our common numbers, without which arithmetic must have stood still.

The word *Azimuth* also came from there; which was explained at p. 141, and is to Altitude what Longitude is to Latitude, and what R. A. is to Declination; viz., the distance between two great circles like meridians, passing through the zenith instead of the pole; and it is reckoned from the north point of the horizon; but the *Prime Vertical* goes through the zenith and the east and west points. *Celestial latitude* is measured from the ecliptic, as terrestrial latitude is from the equator. Formerly the longitude of the planets was, and in some almanacs still is, given by the signs, as ♎ 25°, or ♓ 6°, but more frequently now by the corresponding number of degrees, 205°, or 336°.

An ecliptic is generally drawn on terrestrial globes, crossing the equator at 30° west and 150° east longitude. But there is no reason for its crossing at one point rather than another, and in fact an ecliptic has no meaning there, except as a ready mode of seeing over what latitude the sun is vertical at any time of the year; and that may be done as well by taking his declination from the almanac, which is the same thing as terrestrial latitude. On the celestial globe it has a

meaning and a proper place; and the great circle half way or 90° from ♈ and ♎, which therefore passes through the poles of the equator and ecliptic and the places of the solstices, is called the ***solstitial colure.***

Orreries—so named after a Lord Orrery who bought a ***planetarium*** 170 years ago—are of great antiquity, and may be called working models of the solar system. In old times of course they made the earth its centre; but they were made by Huyghens and Romer to illustrate the Copernican system and the motions of Jupiter's moons, and much later by Dr. Young and other astronomers. By a proper combination of wheels, of which I gave a simple illustration at p. 77, they can be made to show all the regular motions of the earth, moons, and planets, the changes of seasons, and the conditions of eclipses. But an orrery with a moon only a quarter of an inch wide, and an earth of an inch, ought to have a Jupiter of 11 inches, a Saturn's ring of 21, a sun of 9 feet, and a platform of 11 miles' diameter to take in all the orbits of the planets on their proper scale.

CHAPTER VI.

ON TELESCOPES.

BOOKS on astronomy generally give some account of the modes of mounting and using telescopes for different purposes, while the explanation of the principles or 'theory' of the telescope itself is left to treatises on optics.* But for that simple looking tube, with only a large magnifying glass at one end and a small one at the other, we should hardly know more of astronomy than the Chaldeans did, or at any rate Copernicus, who only guessed but could not prove that the sun is the centre of the solar system; nor should we be able to measure a single celestial distance, or even the earth itself accurately, or navigate the great seas with certainty of reaching any given place, and the civilization of the world would stand still.

I said at p. 43 that Galileo was the first inventor of telescopes; and so he was for astronomical purposes, though it appears that they had been made above three centuries before in England by the famous Friar Roger Bacon, already mentioned at p. 159, who was the greatest philosopher from his own time to that of his more celebrated namesake Francis Bacon, Lord Verulam and St. Albans, commonly called Lord Bacon,

* There is a clear and concise treatise on optics in Chambers's Educational course, requiring very little knowledge of geometry.

which he never was. Friar Bacon also invented gunpowder, and so many other things that he was accused of magic, like Dr. Faustus one of the inventors of printing, and imprisoned by the Pope for ten years—much longer than Galileo, and only released when he was 74, after which he lived six years, and died at Oxford in 1294. But the invention perished with him, and was lost till it was made again by Jansen and Lippershey in Holland, in what form is not known, and very soon afterwards by Galileo in 1609, in the form which still survives in opera-glasses; which, as everybody knows, are always made double or binocular; and it is remarkable that the early astronomical telescopes of other kinds were generally double, which has long ceased to be the case.

That was however soon superseded by what is called the 'astronomical telescope,' as the best kind of watches are called chronometers, though all watches are or profess to be chronometers or time-measurers. Kepler seems to have suggested this construction first, though he did not execute it, nor perceive all its advantages. One Father Scheiner was the first who did it, and it was afterwards improved by Father Rheita; and still more in 1656 by the celebrated Huyghens, who discovered the true law of refraction, and also invented a compound eye glass which is still for some purposes the best. For the present I only say that Galileo's telescope is a short tube with a large convex lens, or one thickest in the middle, at one end, called the object glass, and a small concave one called the eye glass at the other end. The 'astronomical' differs from it in having a convex eye glass at the end of a longer tube. In both the object

glass must be flatter than the eye glass to produce any final magnification: indeed the amount of it depends entirely on the proportion of the curvature of the eye glass to that of the object glass. We shall see that its action is quite different from a single magnifying glass.

Space-penetrating power.—But before we talk of magnifying there is another part of the business of telescopes, which is much more simple, though less understood by most people, who are satisfied with knowing that a telescope is a combination of glasses for making distant things look nearer by magnifying them. It is not by means of magnification that telescopes enable us to see millions of stars beyond what the eye alone can see: or as it is called, to penetrate farther into space. The *space-penetrating power* depends simply on a telescope being a very large eye, and bringing into our eye as many more of the rays from each point of a distant object as the area of the object glass exceeds the area of the pupil of the eye. It is true that a telescope cannot bring into the eye all the rays it receives unless the magnifying power is at least equal to the width of the object glass divided by the width of the pupil; but the magnification of powerful telescopes far exceeds that.

To measure that power we must remember that we should want an eye four times as *large*, or twice as *wide* to see a star just visible now, if it were removed twice as far; since the rays are spread out four times as thin at twice the distance. Besides that, at least one eighth of the light is lost in passing through the glasses; which is the same as if the area of the object glass

were only $\frac{7}{8}$ or its width $\sqrt{\frac{7}{8}}$ or ·935 of what it really is. Therefore the space-penetrating power of a refracting telescope is to that of one eye as ·93 of the width of the object glass is to the width of the pupil—nearly a quarter of an inch; or it varies, not as the area, but as the width of the glass.

By far the largest telescopes that are made have concave mirrors instead of object glasses, as I shall explain afterwards; and their space-penetrating power would be greater in the same proportion as their width; but much more light is wasted by reflection than by passing through glasses: indeed nearly half generally instead of one eighth. Therefore the space-penetration of Lord Rosse's telescope, whose mirror or speculum is 6 feet wide, is $\frac{72}{\sqrt{2}} \times \frac{4}{\sqrt{2}}$ or 144 times that of a pair of eyes; for the power of two eyes is $\sqrt{2}$ times that of one, on the same principle as just now stated. This means that the telescope will penetrate a sphere of stars of 144 times the radius that we can see without it; and therefore, if the stars were scattered equally thick all through what we can only call infinite space, we should see 144^3 or nearly 3 million times the usual number with such a telescope. In some books it has been put much higher; but they neglect the loss of light, and compare with one eye only. The largest refracting telescopes yet made have been—one by the late Thomas Cook of York for Mr. Newall, with a glass of 25 inches, and a still larger one by Alvan Clark for the United States Naval Observatory in 1873, with a $27\frac{1}{2}$-inch glass having a clear opening within the rim of 26 inches, and a focal length of 30

feet. The penetration of that would be about 67 times a pair of eyes by the above mode of calculation, or would see above 300,000 times as many stars.

Magnification.—In both cases the whole of the rays received by the telescope are made to converge by the mirror or object glass in just the same way as the rays from the sun converge into a small bright spot or image of the sun in a common burning glass. But this is connected with the other part of the business of telescopes, which requires a good deal more explaining. Magnification, either by a single convex lens or by any combination of glasses, consists in the rays which come to the eye from the outside of an object being made to come at a wider angle with each other than they do naturally. But this is not all: if it were, a common 2-inch magnifying or burning glass which makes a bright spot at 3 inches from the glass would magnify the sun from 32′, its apparent diameter to the naked eye, to 37°: that being the angle at which the outside rays converge to a focus through such a glass.

Indeed if you could look at the sun through such a glass with your eye at the focus without burning it out —as you may at the moon—it would appear magnified enormously; but then it is all in confusion, because the rays from each point of the sun or moon are not brought to a point again at the back of the eye, where a distinct (though inverted) picture of everything seen must be formed if it is to be seen distinctly. The picture of the last thing seen actually remains there for a time after death, as Scheiner discovered long ago. The condition for distinct vision is that the bundle or *pencil* of rays which come from *each* point of the object to the

glass shall be brought to the eye in a parallel state, like a thin stick of straight wires (or rather diverging for short-sighted eyes), but the pencils from the *various* points of the object must make some sensible angles with each other if it is to have any apparent size; and the wider those angles are the more it is magnified.

It may occur to you to ask what is the use of applying telescopes to the stars when no magnifying power will make them look any larger (p. 322). The use is to measure their distances from each other and from the meridian or the equator, or any other circle; for those distances are magnified by the telescope, and are measured by the time the star takes to move over them, or by the angle which the telescope has to be moved through from one star to another.

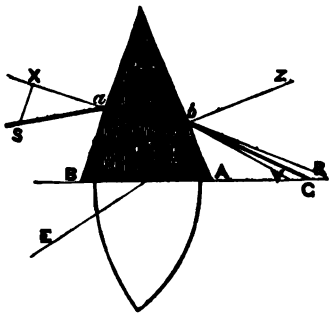
Again you may say that terrestrial objects seen through a telescope do not appear larger than usual but only nearer. But that is exactly the same thing; for, as I have said several times, the apparent size of anything has no meaning except with reference to its distance; and it is only because we know by experience the size of houses, trees, and men, that we do not think of them as appearing larger when seen near than far off. When we come to mountains or buildings above the usual size we are constantly deceived, and generally take them to be smaller and nearer than they are. On the other hand very small things do appear very large through a microscope, which is only a telescope adapted for near objects, because we can never see them so large with the naked eye, in consequence of the eye not being able to see anything distinctly nearer than 5 or 6 inches; for then the rays from each point diverge

too much to be brought to points again by the lenses of the eye, and to form a distinct picture on the retina.

The short explanation of an astronomical telescope is this: see the figure at p. 359. Rays of light spread from the point A of the object ♀ in all directions. A certain number or *pencil* of them is received by the object glass OPQ all over it: after passing through that convex lens they converge to a point or focus *f*. In like manner the rays from another point B come to the object glass, crossing the A rays without at all interfering with them, and then converge to another point *i* at the same distance as *f*; and in that way an inverted image *fi* of the object AB is formed there, which is as much smaller than the object as O*f* is less than OA, and very near the eye glass *abc*. Then the rays that came from A diverge a little from *f* to *a*, and the B rays to *b*, and there both pencils are refracted and made to come, each in a parallel state, into the eye at E, but making a much wider angle *a*E*b* with each other than they did coming from the object: which is the increase of apparent size or magnification. For the rays in each pencil are changed by the eye glass from diverging into parallel ones, which is the state the eye requires for distinct vision.

Law of refraction.—But though this is enough as a general description of a telescope, I have given no reasons yet for anything that the lenses do, nor explained what the magnifying power is for any given pair of glasses. If you want to understand these things we must begin again with refraction, and see how they all come from one tolerably simple law—not to go farther back into the causes of that, which are far more

complicated. For this purpose we must use a figure.



ABC is a prism or wedge shaped piece of glass, and S*a* a ray of light entering it at *a* from the air, and X*a*Y perpendicular to the surface. Then instead of the ray going on straight, it is bent aside at *a* into the direction *ab* nearer the perpendicular, unless it happens to be itself perpendicular, in which case only it goes straight.

So long as the obliquity is small, as it always is with the rays which fall on telescope lenses, the angle S*a*X in the air may be considered to bear the constant proportion of $\frac{3}{2}$ to the angle *ba*Y in the glass: which number is therefore called the *index of refraction* for glass; as $\frac{4}{3}$ is for water (compared with air) and 2·44 for diamonds. At *b* the ray emerges again out of glass into air, and follows the same law, turning off into an angle Z*b*G which is $\frac{3}{2}$ of the angle *ab*Y in the glass, Y*b*Z being the perpendicular at the surface. The angle in the thinner medium is generally called the 'angle of incidence' and that in the denser one the 'angle of refraction;' but they are just the same whichever way the ray is going, and it is better to think of them as the air angle and the glass angle.

If the two surfaces of glass were parallel, it is evident that the second refraction at *b* would just balance the

first at *a*, and the ray would emerge parallel to its first direction, only a little pushed aside, according to the thickness of the glass and the obliquity of the incidence on either surface. But if the surfaces are inclined, the emerging ray (whether it is S*a* or G*b*) deviates from its original direction, as you see in the figure, and is brought downwards towards the thickest part of the glass, or away from the angle C, called the *refracting angle*, where the two refracting surfaces meet. It may be proved by a little geometry that the deviation is half the refracting angle of the prism ABC; i. e., if S*a* is parallel to BAG or any other line in that sort of position, the angle *b*GA will be half the angle C, provided the angles at *a* and *b* are small ones; and they cannot be both small unless C is.

Though this is not a treatise on optics, yet to prevent mistakes I had better tell you that when the angles of incidence and refraction are not small the relation between them is not quite so simple. If you draw any line SX at right angles to *a*X, the fraction $\frac{SX}{Sa}$ is called the *sine* of the angle S*a*X, as already stated in effect at p. 276. And the true relation between the two angles, either at *a* or *b*, is that the sine of the air angle is $\frac{3}{2}$ of the sine of the glass angle. You will find in books of mathematical tables the sines of all angles up to 90°, where $\frac{SX}{Sa}$ evidently becomes 1, and the sines can go no farther, but decrease again; but so long as the angles are small, the sines increase in very nearly the same proportions as the angles themselves, which simplifies telescope calculations materially.

Internal reflection.—On the other hand when you get up to 41° 48′, the sine of that is $\frac{2}{3}$; and therefore no glass angle beyond that can have any corresponding air angle, as there is no angle with a sine greater than 1. And then comes the remarkable consequence, that a ray trying to emerge from glass at any greater obliquity than that, which is called the *critical angle*, cannot get out there at all, but is reflected back again, more completely than if the back of the glass were silvered for a mirror. Thus if the angle *ab*Y in the figure were as much as 42°, the ray would not go out of the glass, but would be reflected to *c* on the third side of the prism, and there go out to E with the usual refraction, unless it arrived at *c* also too obliquely to get out, when it must try again at the side BC. The critical angle of water is 48½°, and of diamonds only 28°: which is the reason of the brilliancy of diamonds, and of dewdrops which stay on certain leaves unbroken, viz., that a great deal of the light falls on them too obliquely to pass through their back surface, and so it is reflected.

Reflecting prism.—Therefore if light falls directly (i. e., perpendicularly) on one of the narrow faces of a prism whose angles are 90°, 45° and 45°, it reaches the wide face at the same angle 45°; and as that exceeds 42°, it is all reflected out directly at the other narrow face, at right angles to its original direction; and such a prism is a far better reflector than a flat mirror set at the angle 45° to reflect at right angles. You will see at p. 377 the use which is now made of this in Newtonian telescopes. A little light indeed is reflected at every change of surface, which is the reason of the loss of light in passing through the glasses of telescopes, but

only a little until the critical angle is reached. You will see how this is turned to account in telescopes for looking at the sun (p. 380).

If you find it difficult to believe that light cannot get through a transparent medium anywhere, you may verify it by hanging something a little under water on the far side of a trough or basin, and looking at it while you bring your eye down to the water on the near side. At a certain distance it will vanish, although you can see the far side of the trough below and beyond it. The reason is that the rays which ought to come from it to your eye reach the surface of the water too obliquely to come out, and are reflected down again internally, as you would see if your head were under the water instead of over it.

Thus far there is no material difference between a thin prism of which ABC is a section and a convex lens; for though its faces are segments of spheres (either alike or not), a ray passes through it exactly as it would through a prism touching it at the points *a*, *b*, as I have drawn the figure. But when many rays fall on different parts of the surface, a lens will manifestly affect them all differently; and those which fall near the edge will be more refracted than those near the middle, because the surfaces are more inclined to each other near the edge than near the middle, where they are parallel. And it fortunately happens that all the rays from from any one point, far or near, after passing through a convex lens, whether directly or obliquely, converge again (very nearly) to another point or focus somewhere in a straight line through the first point and the centre of the lens; which line is called the axis of

that pencil: as you may see with the rays spreading from A or B over the lens OPQ in the astronomical telescope at p. 359, and converging again to *f* and *i* in the axis of each pencil.

Focal length.—If the rays come from a point so far off that they are practically parallel, the point of convergence is called *the* focus of the lens, and its distance from the lens is called the focal length. Conversely if rays emanate from a point at the focal length they will emerge from the lens parallel; but if from a point nearer, they will not converge at all, but only diverge less than before: if from a point beyond the focal length, they converge to a focus somewhere beyond the focal length on the other side. These two points of divergence and convergence are called *conjugate foci*, and are related by a very simple rule: if the distance of one is u and of the other v and the focal length f, then v is the product of fu divided by their difference, u being greater than f: if it is less, then v is the distance beyond u from which the rays appear to diverge after passing through the glass.

The focal length of a glass lens bears a very similar relation to the radii of curvature of its two surfaces. For it is twice their product divided by their sum; or by their difference, if one side is concave but the lens effectively convex, or thickest in the middle, which is called a *meniscus*. Therefore the focal length of an equi-convex lens is the radius of curvature of each face, and of a plano-convex it is twice the radius of curvature of the one spherical face. Another practical rule is that the focal length is the square of half the width of the glass divided by its effective thickness or excess of

the middle over the edges. But these rules only hold for glass, whose refractive index is $\frac{3}{2}$.

Concave lenses also have a focal length, which may be calculated by the same rules, but it means a different thing. When parallel rays fall on such a lens they are made to diverge as if from a point on the same side as the object, and the distance of that point is called the focal length. Rays really coming from that point, or any other, by no means emerge parallel, but are spread out wider by a concave lens.

A pencil of rays may pass either 'centrically' through a lens, like the pencil AOf and all the others through the object glass of the telescopes (p. 359), or 'eccentrically' as they do at a and b in the eye glass. The surfaces across the centre of a lens are practically parallel to each other, or perpendicular to the axis of any pencil of rays through the centre; and therefore centrical pencils are not refracted at all, though the rays which compose them are refracted to a focus somewhere in the axis of the pencil. But eccentrical pencils are wholly refracted, and bent inwards, and so made to converge to the eye E beyond the lens.

That also shows how a common magnifying glass or convex lens magnifies things near the focal distance. All the points in the object, except the middle one, are seen by eccentrical pencils, which are refracted and made to converge to the eye at wider angles with each other than the pencils by which those points are seen without the glass. Without the glass the apparent size of ♁ would be fEi (drawing lines fE, iE to the eye); with the glass it is aEb, a much larger angle.

If the object is much nearer the glass than its focal length, the rays of each pencil diverge too much for the glass to make them parallel enough for distinct vision; and besides that, the angle *fEi* is then nearly the same as *a*E*b*, and so there is little magnification. On the other hand, if it is too far off, the rays of each pencil converge into the eye and again produce indistinctness; and the pencils also become so nearly central that there is no sensible magnifying. The magnifying power of a glass for objects near its focus evidently varies inversely as the focal length.

Images.—But a magnifying glass has another and a very different effect, much less known, on which the action of a telescope depends. If you fix a common long-sighted spectacle glass in the open window, and recede from it, any object which you see through it becomes more and more confused, until it suddenly reappears inverted and smaller. In fact it is no longer the object itself that you see, but its image, formed as I described at page 349. If the object is so far off that the rays of each pencil may be called parallel, the image will evidently be at the focus of the lens; and smaller or larger according as the focal length is less or greater; for the size of the image is to the size of the object as the focal length is to the distance. If the object is only a little beyond the focal length, the image is farther off and larger, as in microscopes. So the transparent pictures put upside down into a magic lantern, a little beyond the focus of a convex lens, cast a magnified image of themselves upright on the wall, or on a wet sheet if you want to see them from the other side. On the other hand the small bright spot under a burning

glass is the image of the sun, taking his shape reversed in an eclipse, or that of the crescent moon.

Spectacles.—Short-sighted eyes require the rays from each point to come in a diverging state, as they do when the object is very near, though the pencils from the different points then come more converging, and so the object looks larger than average eyes can ever see it. Therefore concave spectacles suit short sight, which make the rays diverge. Long-sighted eyes require parallel rays, and therefore cannot distinctly see things near, without convex glasses to make the diverging rays parallel, not really to magnify the objects, though they do so. Holding a book far off makes the rays of each pencil come parallel or nearly so, but it also makes the letters appear smaller and send less light into the eye; so that it is the same as reading smaller print by a worse light; therefore it is better to use spectacles. But old eyes also require more light than young ones, besides the alteration of the focus, and therefore stronger spectacles for candlelight. It is odd that short-sighted eyes do not become mean-sighted with age, though mean-sighted ones almost always become long-sighted. It is still more odd that they occasionally recover at a great age.

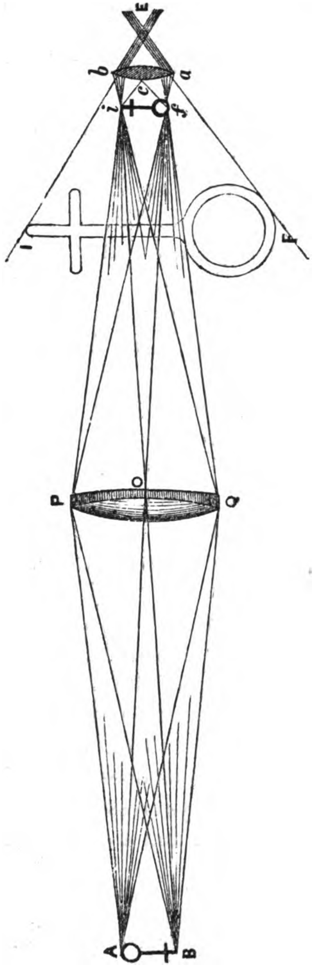
When we talk of parallel rays, it always means the rays of one pencil and not the rays of different pencils; only the stars are so far off that the pencils of rays from their outsides come to us at no angle that can be measured by the utmost magnifying power, and therefore practically as parallel as the rays of each pencil; and that is why a star in the largest telescope looks nothing but a point, though Neptune or the satellites of

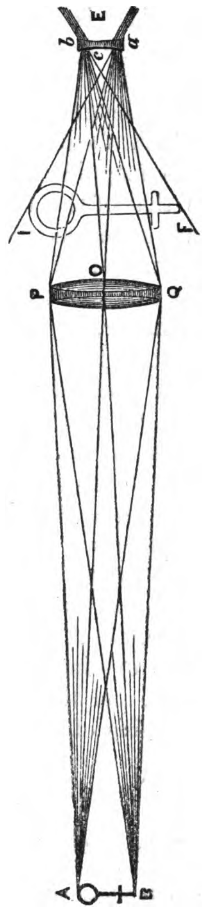
Jupiter can be magnified into discs of visible size. For the rays of a pencil diverge no more than the width of the *glass* divided by the distance, while the extreme rays or pencils from a planet make an angle = the width of the *planet* divided by its distance.

Astronomical telescope.—You can now understand the operation of an astronomical telescope, and how its glasses must be placed, and how much it magnifies. The image being formed at the focus of the object glass differs from an ordinary small picture in the fact that the rays can only go from it in the directions in which they are sent by the object glass. Consequently the eye glass must be exactly at the proper place to receive them and bring them in a proper state for vision to the eye; that is, for ordinary eyes it must have its own focus coinciding with the image or the focus of the object glass, and be moved nearer to it for short-sighted eyes which require diverging rays.

Magnifying power.—Then for the magnification: the pencils aE, bE are parallel to fc, ic, the lines from f and i through the centre of the eye glass, because f and i are at the focal distance; and so the apparent size of the image $a\mathrm{E}b = fci$, instead of AOB the naturally apparent size of the object, which is evidently the same as $f\mathrm{O}i$. Those angles being small are inversely as the distances of the image fi from O and c (the angles are immensely exaggerated in the figure for distinctness); or the magnification is the focal length of the object glass divided by that of the eye glass. And that is the case in all telescopes, except two of the reflecting ones which I shall mention afterwards.

You see then that the business of the object glass is


E
b
a
c
i
f
I
F
O
P
Q
A
B


E
b
c
a
I
F
O
P
Q
A
B

not to magnify the object, but to form an image of it to be magnified by the eye glass; and the flatter the object glass is the larger is the image that it forms. The average apparent diameter of Jupiter being nearly 40″ or ·0002, the size of its image from a glass of 10 feet focal length is ·024 inches; and with an eye glass of 1 inch focal length, the apparent diameter of ♃ will be magnified 120 times, or into 80′, or nearly 3 times that of the sun or moon, and Jupiter will look 7 times as big as the moon, and Saturn nearly as big as the moon. But magnifying power is reckoned by diameter and not by disc, which varies as the square of the diameter.*

Galileo's telescope (see figure at p. 359) has a concave eye glass placed at its own focal length before the focus of the object glass instead of behind it. Therefore it makes the rays of each pencil come out parallel, but the pencils themselves diverge into the eye at an increased angle *bca*, instead of converging to E. But that produces a widened image in the eye just as well. Here no image is formed in the telescope, and there is no inversion; for though the pencils A*a*, B*b*, cross at O as before, the A pencil from the top of the object comes into the eye in the direction I*ca*, as from the top of a larger ♀, and the B pencil F*cb* comes from the bottom. The crossing of the apparent directions does not signify. In the astronomical telescope the rays cross in a different way at E where they enter the eye, and a *diaphragm* or

* This does not apply to advertisements of microscopes magnifying drops of water many thousand times; they evidently call a magnification of two hundred, 40,000: possibly they even reckon the insects in a drop of water by cubical size.

plate with a hole in it is fixed there to guide the eye to the proper place.

Field of View.—But the capacity or field of view in the Galilean telescope is limited by the size of the pupil of the eye, as the pencils diverge into it; and for the same reason very high magnifying powers cannot be used. I have drawn all the parts of both telescopes out of all real proportion, or else the small angles made by the rays would not be visible. The field of view, or the largest apparent diameter that can be seen at once, is evidently the angle *aOb* or AOB, or the width of the pupil divided by the length of the telescope, even if the eye is close to the eye glass: which is a reason for keeping them short. In the astronomical telescope that angle is not the width of the eye, but the width of the eye glass, divided by the length of the telescope, measuring by the axes of the two extreme pencils, which however allows half of their rays to slip over the edges of the eye glass. And the field is limited to that by a diaphragm or plate with a hole in it fixed across the focus to stop any pencils more oblique than those whose axes fall just within the eye glass. Glasses of higher power or shorter focal length are also smaller, and so the field of view is less the higher the power that is used in the same telescope.

Long telescopes with high magnification often have a *finder* set upon their back, which is a short and small telescope of less power, in which the eye glass is much larger in proportion to the length, and therefore the field of view greater. This also was an invention of Newton's.

Another serious defect of the Galilean telescope for

astronomy is, that, as the rays never reach a focus, there can be no cross wires set there to mark the transit of a star, or to define its exact position, which is a most important part of the business of telescopes. The inversion by the astronomical telescope does not signify. In land telescopes of that construction another lens, or more, is introduced to reverse the image back again, and then it is far better than the Galilean.

Brightness.—Although a telescope gathers much more light than the eye, and would act as a burning glass, it does not make objects look brighter, but dimmer. For if all the light received by it enters the eye, it can only exceed that received without the telescope as the area of the object glass exceeds that of the pupil, or as the squares of their widths. And if you draw a pencil of rays going centrally through both glasses you will see that its width at the object glass (which is the width of the glass itself) is to its width at the eye glass (and therefore to that of the pupil, if the whole pencil is to enter it) as their focal lengths, or as the magnifying power. But the rays are spread out wider and thinner over the magnified object or image in proportion to the square of the magnifying power, and therefore (under the above condition) as the area of the object glass is to that of the pupil. So the magnification diminishes the brightness at least as much as the size of the object glass increases it; and the brightness is reduced still more, either if the pencil does not fill the eye by reason of the magnification, or if it is too large to be all taken in. Besides that, there is the loss of light in the glasses, as stated at page 346. So the image in a telescope can never be as bright as the object.

Colour dispersion.—But both kinds of refracting telescopes, so far as I have described them yet, have a serious defect, which made Newton and others abandon them for reflecting ones. If you look through a strong magnifying glass, or a common cheap telescope, you see things fringed with colours and indistinct; as is still more visible in the magnified images of a magic lantern. The reason is that the seven colours which make up a ray of white light are not refracted equally by any substance; or every ray of white light is split up by refraction into seven coloured ones, some of which are more refracted than others. It is not very easy to see it at one refraction, but it is evident enough with two, as when a ray emerges from a prism. Turning back to p. 350, the whole emergent ray does not come in the direction *b*G, but only the middle or green part of it, while the red is refracted through the smaller angle Z*b*R, and the violet as much as Z*b*V: the whole spectrum consisting, first of invisible cool and chemically acting rays, most refracted, then violet, indigo, blue, green, yellow, orange, red, and then invisible hot rays, as described at p. 105. Those at the violet end are comprehensively called blue, and those at the other end of the spectrum, red.*

Consequently a lens cannot bring any pencil of rays really to a focus; but a blue image of every point of the object is formed nearer to the lens than the red one, and a green one somewhere between the two, which is overlapped by the red rays before

* We are not concerned here with the fact that all the seven prismatic colours can be made up by combining different shades of red, blue, and green, not red, blue and yellow, as in artificial colours or paints; for yellow is really green and red (see Sir J. Herschel on Light, 'Familiar Lectures,' p. 258).

they reach their focus and by the blue diverging again after they have passed their focus. Therefore the smallest image of each point in the object is a little circle about half way between the red and the blue foci; and that circle is never less than one fiftieth of the width of the lens. Consequently large object glasses could not be used until this defect was cured by the invention of compound *achromatic* or colourless glasses, which I will describe presently.

Moreover the colour dispersion is increased to the eye by increasing the magnifying power of the eye glass, or using one of very short focal length; and as the magnifying power of the telescope is the focal length of the object glass divided by that of the eye glass, the only way of getting a highly magnifying telescope was to have an object glass of very great focal length. Accordingly they were sometimes made from 100 to 600 feet long, so that the object glass had to be set upon a pole, without any tube, and with strings and levers to pull it into the right direction for sending the rays into the eye glass. Such telescopes were called aërial. It is curious that we are returning to them in one respect; for tubes are being superseded by open frames for very large telescopes, but the glasses are fixed in them.

Achromatic telescopes.—By an unlucky mistake in an experiment Newton missed the discovery, which was not made use of till nearly a century afterwards, by Dollond the optician, though it was made before by Mr. Hall of Worcester, that glasses of different densities have different powers of dispersion into colours for the same amount of general refraction. A lens of flint

glass, which contains lead and has a specific gravity 3·6, spreads the blue rays farther away from the red, though it may refract the red or the green equally with a crown glass of the same convexity, whose specific gravity is only 2·5. Therefore a slightly concave flint lens may refract the blue rays outwards just enough to correct the excessive refraction of them inwards by a very convex crown lens, while it leaves a general balance of refraction over, because the two lenses together are on the whole convex or thickest in the middle. Or the same thing may be done by putting a double concave flint lens between two convex crown lenses. I have drawn the object glass of the Galilean telescope in this way, and that of the astronomical with only two glasses, in the pictures at p. 359.

The rule for determining their proportions is simply that the focal length of the flint glass must bear the same proportion to that of the crown glass (or of the pair of them) as their respective dispersive powers, or as ·068 to ·033. For the width of the spectrum of a flint glass prism, or the angle between the extreme red and violet rays, is ·30 of its mean angle of refraction, or the refraction of the green ray, while the ratio of dispersion to mean refraction by a crown glass is generally ·17, or about half as much as by the flint glass.

Secondary colours.—Still the correction is imperfect, by reason of what is called the *irrationality* of dispersion; which means that the ratio of dispersion to the mean refraction is different for the different colours by any two kinds of glass. A flint glass prism which disperses the blue rays as far from the red as a given crown glass (of a greater refracting angle) does not

disperse the green rays quite as far from the red as the crown glass does. Consequently when the prisms are put together, turned opposite ways, they bring out the red and blue rays together, but leave a little green behind; and so the image through a compound lens of that kind is still fringed with green, and has not a sharp outline; or the *definition* is imperfect. And although this secondary dispersion is only a 60th of the primary, and therefore a 3000th of the width of the object glass, it is enough to leave a sensible advantage on the side of reflecting telescopes, which make no colours. Even if a third glass of some different composition is used, none has yet been discovered which completely annihilates the colours.

Dr. Blair indeed did effect it in a temporary way by fluid lenses (of course enclosed between glasses), which had the requisite action on the different colours: but they could not be made to last. Certain oils and gums refract green light much less than even flint glass; and a long table of them, in the inverse order of their effect on green light, is given in Sir D. Brewster's treatise on Optics in the Encyclopædia Britannica. Mr. Justice Grove above 20 years ago made solid lenses of resin and castor oil,* and also of hardened Canada balsam and castor oil, interposed in a meniscus form between the crown and flint glass lenses, which he said very nearly destroyed all secondary colour; and he advised opticians to pursue the subject on that footing, as they alone have generally the means of doing it. Mr. Wray, an optician, patented a 'semi-fluid cement' of oil of cassia and castor oil, or Canada balsam, to be interposed as a

* See R.A.S. 'Notices' of 1853, and 'Phil. Mag.,' March of 1867.

meniscus lens 'hermetically sealed' between the crown and flint lens, the crown glass being made flatter at the back than usual, to leave room for the meniscus of cement, as Sir W. Grove's was.

This is said to answer extremely well for the achromatism, but great doubt is expressed as to the possibility of making any but a solid composition permanent, under the changes of temperature to which telescopes are exposed. For an observatory must keep nearly the same temperature as the air outside, or the refraction is disturbed (p. 277). The glasses of a compound object glass are indeed often united by a transparent cement of the same refractive index as glass, as nearly as possible; since that prevents internal reflection (p. 352), by making the two glasses as it were continuous, instead of leaving a film of air between them. But such cement is of the same thickness throughout, and as thin as possible; while these dispersive cement lenses are thickest in the middle, and therefore do not expand and contract equally at the middle and the edges, and are difficult to keep hermetically sealed between glasses if they are fluid. The principle of these inventions is, that the oils and resins refract green light less than glass; and therefore if the convex lens is in effect made partly of glass and partly of these things, they produce together a convex dispersion of all the colours in such a ratio that the concave dispersion of the flint lens can correct it: or that the two opposite dispersions become 'rational' instead of irrational.

Spherical aberration.—Independently of colour dispersion, a lens with spherical faces does not bring the rays of any pencil or of any colour absolutely to

a point; for those which come through the edges are brought to a focus rather sooner than those near the middle of the lens. The amount of this spherical aberration, or *astigmatism* (not coming to a point), as it is also called, depends on the shape of the lens, and even on which side of it faces the light, if the two sides are different; and it is always much less than the chromatic dispersion of a single lens. The shape which gives the least aberration (of those which can be used) has the front face 6 times as convex as the back; which opticians call a *crossed lens.* The diameter of the least circle of aberration, or the image of a point in such a lens, is only the 2160th of its width, when the focal length is 10 times the width. And it increases as the cube of the width, but inversely as the square of the focal length: so in that respect also the long aërial telescopes had an advantage.

Theoretically this defect may be cured by making the faces of the glass not quite spherical, but practically they cannot be ground in any other form. And fortunately the aberration can be corrected in another way, by selecting proper degrees of curvature for the separate lenses of a compound object glass, without interfering with their achromatism, as that depends only on their focal lengths, not on their total convexity and concavity. Various distributions of the curvature will answer, and Professor Pritchard calculated a table of corresponding curvatures to suit various focal lengths, so that any glasses may be matched at once. The best general forms are like those in which I have sketched the two object glasses at p. 359 with the curvatures very much exaggerated. The crown glass in front has its front

face more convex than the back, and the flint glass behind fits it, and has its back convex, but less so than either of the crown glass faces, so as to be effectively concave (which is called a 'reversed meniscus') while the whole object glass remains effectively convex. There are similar arrangements for treble glasses.

Compound eye pieces.—Hitherto I have spoken of the eye glass as a single lens: which it still is for very high magnifying powers. But where distinctness or *definition* is chiefly aimed at, the eye glass also must be corrected for colour, for spherical aberration, and for something else. For suppose a centrical pencil of rays to be brought exactly to a focus by the object glass: it goes on to the eye glass eccentrically, except of course the one pencil which keeps the axis of the telescope; and then instead of being refracted into a round and colourless stick or pencil of parallel rays, it is spread out into a coloured oval one, of rays not parallel in one direction (that of the plane of the paper in the figures at p. 359) but diverging. Moreover all the points of the image ought to be at the focal length of the eye glass, or it ought to be concave towards the eye glass: but in fact it is rather concave the other way, being formed by the object glass. Consequently the rays from the outside of the image come from rather too far off and are more turned inwards by the eye glass than if they came exactly from its focal distance.

It is found that the best way to correct all these errors is to divide the work of the eye glass between two less convex lenses separated by a space equal to the mean of their focal lengths, or 2 inches if the focal

length of the first is 3, and of the second (next the eye) 1 inch. The first is called the field glass, because the field of view now depends on that (see p. 361), and it stands at half its own focal length before the focus of the object glass; so that the image is now formed between the field glass and the eye glass. The best shape for them is a meniscus field glass with radii of curvature as 4 to 11, and a 'crossed lens' (i. e., of radii 1 and 6) for the eye glass, both with their convex faces towards the light.

This is Huyghens's eye piece, except that he had two plano-convex lenses; and it is also called the *negative* one, because the image is behind one glass, but before the other. And this involves a serious defect for some purposes. The image no longer has the same proportions as the object, but the outer parts of it are contracted, because the pencils near the edges of the field glass are most refracted or bent inwards. The image of a square would thus have its corners bent in or the sides curved out, and a piece of netting would appear to have the outer meshes smaller than the middle ones. Consequently it will not do to put cross wires there for measuring an object or its position.

For that purpose the *positive* eye piece was invented by Ramsden, a famous instrument maker about 1780. In it both the glasses are beyond the focus of the object glass, where the wires are, and so they and the image are equally magnified by the eye piece. The glasses are two equal plano-convex lenses with their convexities facing, at a distance = $\frac{2}{3}$ of each of the focal lengths; and the field glass is a quarter of its own focal length beyond the focus of the object glass. It is not quite

achromatic, but it has less spherical aberration and distortion than the other.

The principle of both the positive and negative eye pieces is this. An achromatized pencil from the object glass reaching the field glass eccentrically, its blue rays are there most refracted, and are sent to the eye glass nearer its centre than the red rays which are least refracted. But those which strike the eye glass nearest the centre, or least obliquely, are least refracted by it; and so the least refraction there balances the greater refraction before, and the blue and the red come out parallel, and then the lenses of the eye unite them into a colourless point. The same kind of compensation takes place also between the rays which are most refracted and least refracted by the field glass by reason of spherical aberration, and the convexity of the image being turned the wrong way; and so these compound eye pieces correct both defects.

The inverted image is turned into an erect one in land telescopes by an eye piece generally made of four glasses; but we are not concerned with them. All eye pieces are made adjustable for different eyes by the eye glass sliding nearer to the object glass for short sighted eyes, which require the rays of each pencil to diverge a little instead of being parallel.

Micrometers.—The measuring wires which I have several times spoken on are stretched across a small frame which slides a little across the telescope with a screw, of which the head is graduated, so that you can measure exactly how far any wire has to move to bring it into contract with a star, to measure its distance from any other star or the meridian or zenith. The same

screw apparatus is also used outside some telescopes to adjust or measure their position, with microscopes attached to it, and in each case it is called a micrometer, or measurer of small distances. The wires are fixed in and parallel to the meridian for transit purposes, and horizontally for measuring altitudes and polar distances. The wires in the reading off micrometers are set so that they may be brought to coincide with the graduations of the circle they are to read off. They are sometimes spider lines, as they are finer than any artificial lines that can be made; and they have to be illuminated by light reflected on them from a lamp.

Since each pencil of rays spreads over the whole object glass, half a lens makes the same image as the whole one, only not so bright. If the two semicircular half lenses slide along their common diameter two images will be visible whenever the two centres do not coincide. And the distance from the first contact of the images to the last, measured by the turns of the screw which moves one half lens along the other, is evidently twice the width of each image. This micrometer is the best for measuring diameters of the sun and planets, and the distances apart of double stars.

REFLECTING TELESCOPES.

In order to escape from the inconveniences of either colour dispersion or long aërial telescopes, Newton and Gregory and Cassegrain and Herschel invented their several forms of the reflecting telescope, which produces no colours, except in the eye glass, which Huyghens had corrected long before compound object

glasses were made. They are still used because mirrors or *speculums* can be made much larger than object glasses in the present state of art; and we have seen that the space-penetrating power of a telescope varies as its diameter. I have already compared the largest reflector and the largest refractors yet made, at p. 346.

The theory of reflection is much simpler than of refraction, and is all deduced from this one law of nature, that rays are always reflected from any surface at the same obliquity as they fall upon it. Consequently an object appears as far behind a flat mirror as it really is in front of it, the apparent distance being determined, as the real distance is, by the different view which each eye takes, or else by moving one eye (p. 123). Therefore you can see your whole body in a mirror half as wide and half as high as you are.

Reflection from curved surfaces is not so simple as from flat ones; but, as with lenses, it is much simplified by the mirrors of telescopes being only small segments of large spheres, and by the rays only falling on them at a very small obliquity. I had occasion to say at p. 302 that the tangent at any point of an ellipse (or a straight line which coincides with the ellipse there for a moment) makes equal angles with the two focal distances SP, HP (p. 305). Consequently by the law of reflection rays of light emanating from one focus of a prolate spheroidal mirror are all reflected to the other. Again, a parabola is only part of an ellipse with one focus at an infinite distance from the other (p. 290); and therefore rays coming from an infinite distance, i. e., parallel rays, are all reflected by a para-

bolic mirror to its focus; and rays from a lamp at the focus are all reflected parallel to the axis. Moreover it may be proved that a parabola for a short distance near the apse coincides with a circle whose radius is twice the distance SA of the focus from the apse, so nearly that if the speculum is 6 feet wide, and SA is 50 feet, the spherical surface at the edge (where the error is greatest) is only the 8000th of an inch beyond the proper parabolic surface. And therefore a spherical mirror reflects parallel rays (not far from its axis) to a point half way along the radius of curvature, subject to the slight deviation of the circle from the parabola.

The diameter of the least circle of aberration or smallest image of a point is the cube of half the width of the mirror divided by the square of 4 times the focal length, or by the square of the diameter of the sphere of which the mirror is a part. So that it is only the 12,800th of the width of a mirror whose focal length is 10 times its width. Lord Rosse found the means of so polishing the speculum as to remove the spherical aberration and make the surface parabolic. The defining power of these great reflectors has generally been considered inferior to that of the best refractors. But the late improvements, especially in silvered glass specula, seem to be removing that inferiority.

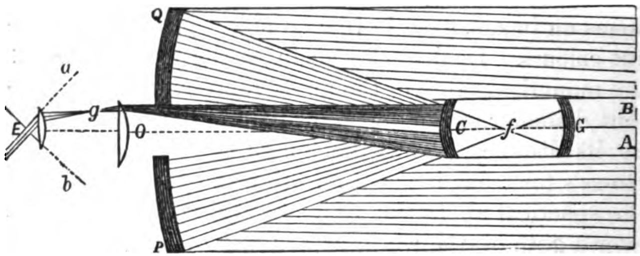
Lord Rosse's speculum weighs 4 tons and is supported at the back on many points by a complicated system of levers to balance the pressure; and so was Mr. Lassell's 4 feet speculum; for an inequality no thicker than a thread bends a heavy speculum enough to destroy all convergence to a focus.

All reflecting telescopes have a concave mirror as

wide as the tube, placed at its lower end, which receives the rays from the object and sends them to a focus, and forms an image, exactly like an object glass, only without colours. Its diameter again = the focal length × the numerical value of the angle representing the apparent diameter of the object, or its real diameter divided by its distance. Thus the image of Jupiter in Lord Rosse's telescope is about an 8th of an inch wide; and the spherical aberration or image of each point is spread over a 20th of the width or a 400th of the area of the whole image, whether it is magnified much or little by the eye glass. You see this is considerably less than the chromatic aberration of a lens only a foot wide with 10 feet focal length, even when it is corrected as much as possible (p. 366). A concave mirror, like a convex lens, magnifies things if your eye is near it, but diminishes and inverts them if you are far enough off to see the image and not the object in the glass: a convex mirror, like a concave lens, diminishes without inverting.

There are four ways of dealing with the rays after reflection from the great mirror. In Gregory's telescope (p. 376) they come to a focus, *f*, making an image there, and then are reflected again by a smaller concave mirror G set in the middle of the tube to another focus *g*, which is also the focus of the eye glass, passing to it through a hole in the middle of the great mirror; and so the first image is reversed back again at the second. In Cassegrain's telescope the second mirror C is convex, and receives the rays before they come to a focus, acting like a concave lens, only sending the rays backwards, less converging than before, to a focus which again coincides with that of the eye glass.

This figure will sufficiently explain the course of the



rays in both these telescopes, especially Cassegrain's, which is the one usually not depicted in other books, while Gregory's is. PQ is the large mirror with a hole in the middle, near whereto is the field glass at O. G is the small mirror of Gregory and C that of Cassegrain, the focus of the great mirror being between them. The field of view in each is the angle which the small mirror subtends at O. In this picture the parallel rays are a single pencil coming from a very distant point A below the axis of the telescope. Those which are not stopped by the back of the small mirror C are reflected to it from the large one, as you see, except that in each case I have given precedence to reflected rays and cut off the incident ones for distinctness. They are reflected from C to the field glass O without yet coming to a focus, which they do between the field glass and the eye glass; and the lenses are so made that the pencil of rays emerges in a parallel state at E but making the angle OE*a* with the axis; or the apparent diameter of an object AB (B being as much above the axis as A is below it) is *a*E*b*, as in the astronomical telescope.

The magnifying power of both these telescopes will be given presently with Newton's.

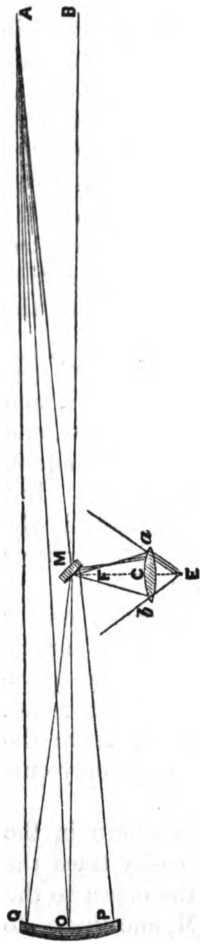
The rays come from the concave small mirror G in Gregory's telescope in much the same way, only they had crossed before reaching it. The small mirror is of course supported by a slight stem from the inside of the tube. Cassegrain's tube is therefore evidently the shorter of the two. His small mirror ought to be of the hyperbolic form and Gregory's parabolic, in order to diminish spherical aberration as much as possible, but Cassegrain's has the great advantage that the aberration of the small mirror tends to counteract that of the large, while Gregory's increases it.

In Newton's telescope the second mirror is a flat one, which makes no difference in the convergence, and also receives the rays before they reach a focus, and might send them to the eye glass through a hole in the great mirror like those other two telescopes. But instead of that, the small mirror is set obliquely in the tube, at an angle of 45° to the axis, and so reflects the rays out sideways at right angles to the axis.

Herschel's telescope dispenses with a second mirror altogether; for the great one is a little askew at the bottom of the tube and sends the rays to an eye piece fixed just within the edge of the tube at the other end. This can only be done in very large tubes, where the observer's head in the mouth of the telescope only cuts off a comparatively small part of the rays.

The action of Newton's telescope is shown in the figure on page 378, in which you may easily trace the pencils of rays from the points AB of the object to the large mirror OPQ and the small one M, and thence to

a and *b*, converging to E the eye at the wide angle *aEb*.



The magnifying power of both Gregory's and Cassegrain's may be said to be the square of the focal length of the great mirror divided by the product of the focal length of the small one and of the eye glass. The magnifying power of Newton's and Herschel's is the same as in the two refracting telescopes, viz., the focal length of the great mirror divided by that of the eye piece.

Sir J. Herschel expected the Newtonian telescope to supersede all others, now that a method has been invented, which Newton himself again 'divined' as possible, of silvering concave glass speculums instead of casting those heavy metal ones which also have to be reshaped every time they are polished. It is not the quicksilvered back of the glass that reflects, as in a looking glass, but a thin film of silver chemically deposited on the front or hollow face, which can be repolished, and even renewed when necessary, the glass being once for all ground to the proper spherical face. It is said that a good speculum of this kind reflects nearly as much light as is refracted through an object

glass, while a bell metal speculum loses one third of the light. The only difference between speculums and bells is that bells are (or should be) made of 13lbs. of copper to 4 of tin, while speculums are 128 to 59, for reasons explained in my book on Clocks and Bells. It is curious that the greater quantity of the softer metal tin makes the speculum alloy harder and more brittle than bell metal, and that again is harder than gun metal.

All the reflecting telescopes but Gregory's make an inverted image at the focus of the eye piece; and they all require compound eye pieces like refracting telescopes. Newton's has this further advantage over all other telescopes, that the reflection out sideways may be made to take place through the hollow cross axis on which the telescope is balanced (if the great speculum is not a heavy one) and so the observer may sit still in one position and has neither to look upwards, which is fatiguing, nor to mount into a box high in the air to look downwards as with Herschel's. Lord Rosse's telescope is Newtonian, but the eye-piece is near the top of the tube. The small mirror is about 6 in. wide in the direction across the tube, and more the other way. The observer stands on a gallery.

Both Gregory's and Cassegrain's telescopes had been almost abandoned because of the quantity of light which they lose by the second direct reflection. Much less was thought to be lost by the oblique reflection in Newton's. But this seems now to be doubted, and it was determined to make the great 4-foot reflecting telescope for the Melbourne Observatory on the Cassegrainian plan. Another great new one however at the Paris Observatory is Newtonian, with a glass silvered mirror.

Helioscope.—There is yet one other kind of telescope which ought to be described now that observations of the sun's surface have become an important part of astronomy. The sun is far too bright to be examined through any common telescope; for though it does not increase the apparent brightness, as explained at page 362, still it acts as a burning glass, concentrating all the light from the object glass into the eye. Consequently it is necessary to reduce the quantity of light very much, and yet keep the other advantages of magnification and of gathering a large pencil of rays from every point. For it does not answer to reduce the light by using a very small object glass, or covering up all of it except a small hole in the middle. It is necessary first to gather a large bundle of rays, and then as it were to filter them by some contrivance which will only let a small part of the light come into the eye. The common expedient of smoked or coloured glass will not do either; for such a glass soon gets heated by the rays which it stops, and is liable to crack; and a coloured image is imperfect, being one in which the other colours are destroyed.

The plan generally adopted is to make the rays fall obliquely on one of the wide faces of a thin prism, a little before they reach the focus. That is very different from sending them directly into a narrow side of a right-angled prism, as in the Newtonian telescope, which is intended to reflect them all internally from the wide face. This thin prism reflects a little of the rays from its first face by virtue of the property which all transparent substances have of reflecting some of the light which falls on them while they transmit all the

rest. Here about one 30th of the rays are reflected, and the rest refracted into and through the prism, and sent away as not wanted through the end of the telescope, which is left open, while the reflected rays come out sideways into an eye piece as in Newton's. The reason for using a prism instead of a flat piece of glass is that the internal reflection from the second surface also would interfere with the reflection from the first surface if they were parallel, whereas the prism sends it off in another direction.

This gathering and filtration of the rays is sometimes carried farther, by using a large double concave object glass, unsilvered, as a reflector of some of the rays which fall on its first surface, leaving all the rest to be dispersed through it into the air behind. The rest of the construction is the same as before. But Foucault discovered that an object glass covered outside with a thin film of silver or gold burnished bright will reflect away the greatest part of the rays, while it is transparent enough to transmit some, which are fit for vision and examination of the sun, but are slightly coloured blue. Probably this helioscope will supersede the others, except that it requires a distinct object glass and practically a distinct telescope, while the first only requires a prismatic eye piece to be used instead of the common one, which is movable.

We have yet to consider the different ways of fixing telescopes for different kinds of observations.

Transit circle.—This most important telescope is mounted on a cross axis lying east and west, so that the tube can only move in the plane of the meridian. It is now made so as to combine the work for which

two telescopes were formerly used, one called a *transit instrument* for observing transits across the meridian, and the other called a *mural circle* because it was set against a wall for observing distances from the equator or the pole. The Greenwich transit circle tube and its cross axis are of cast iron, instead of brass as usual. The pivots of its axis until lately were always set on what are called Vs, being bearings of that shape; for a pivot cannot be made to lie steady in a semicircular bearing. But at last this rude method of obtaining a steady bearing has been superseded by the simple device of cutting a wide notch or piece out of the bottom of a semicircular block, so that the pivot still presses only on the sides, but with a much wider bearing than on Vs, which the axis only touches in a line on each side and often wears away unevenly.

Moreover the new compound of copper and aluminium, which may well be called *al-bronze* (as bronze is a softer bell metal), is coming into use for telescope bearings, as it is four times as strong as brass. It was hoped that it would do instead of silver bands or circles for graduation; but though it spoils with the air less than brass, it is far inferior to silver. It is evidently essential to accurate observation that the bearings should be not only level, but as firm as possible, and they are consequently laid on piers built deep into the ground. The instrument is used to observe the transit of a star across the meridian, by pointing it to the proper height to catch the star, and then looking at it as soon as it comes into the *field of view*, and observing the time by the clock when the image of the star crosses the middle micrometer wire, or rather all the wires in

succession. The difference between the times of two stars crossing the meridian is the difference of their right ascension, after all the necessary corrections have been made.

Again, distances from the zenith are found by measuring the angle which the telescope makes with the vertical when a star on the meridian is seen exactly on the wire which is set across the focus horizontally, or at right angles to the transit wire. And that is done in a remarkably neat way, by first looking at the star itself, and then at its reflection in a basin of mercury which forms an artificial horizon: sometimes a plate of glass is laid upon it to stop any tremor of the surface and to keep it clean. By the law of reflection (p. 373) the telescope always makes the same angle with the horizon when pointed at the star and at its reflection, only in one case it is looking upwards and in the other downwards. Therefore the altitude of the star, or 90° minus the zenith distance, is half the angle moved through by the telescope between the two observations. And the declination, or distance of the star from the equator, or what is more commonly used, its north polar distance (N.P.D.) is easily got from its zenith distance and the colatitude of the observatory (see p. 340).

For this purpose the transit circle has a large wheel fixed to one end of its cross axis with a graduated rim, and the graduations are read off by several *micrometers* (p. 371) fixed at convenient places on the pier near the rim of the circle. In the old *mural circles* the graduated circle is fixed on the wall, and the telescope carries an index, with micrometers for reading off accurately. The telescope need not be so accurately in the meridian for

observing N.P.D. as for transits; and so setting it against a wall as near N. and S. as possible was enough without all the precautions which a transit instrument requires to set and keep it right. The bearings must be exactly level and of the same size, and their centres exactly east and west, and the pivots which ride in them exactly round and exactly alike, and the cross axis exactly at right angles to the telescope axis, or the line through the centres of all the glasses. These things require constant watching and testing, and mathematical formulæ have to be calculated for the necessary corrections. Most of them can be tested by lifting the axis over and reversing the pivots, and looking through the telescope opposite ways; but for heavy cast-iron tubes other methods equally accurate are used at Greenwich. These details however are beyond the scope of this book.

Altazimuth.—A transit or a mural circle being only able to move in the meridian can only see stars when they come close to the meridian. But if the horizontal cross axis of the telescope is carried by a frame which itself turns round on a vertical axis, or on a horizontal bed, it can be directed anywhere; and it will measure altitudes above the horizon by moving the telescope up and down, and azimuths by turning the frame; and so that is called an *Altazimuth.* The level plate at the bottom of the frame is graduated, and there is a fixed index with micrometers against it to show the degrees of azimuth through which you turn it.

The *zenith sector* is only another form, or rather a part of the same instrument, being a telescope moving only over a moderately small graduated arc fixed at the zenith for observing stars near it, and therefore taking

less room for its size than a complete altitude instrument with its vertical circle of degrees.

In the R.A.S. 'Notices' of Dec. 1869, Mr. Carrington said that he had adopted in his observatory at Chart 'Steinheil's principle of making the horizontal axis of his principal telescope the effective optical axis,' by placing a mirror made of a right-angled prism in front of the object glass, so that the telescope need never be raised from the horizontal position; for the mirror formed by the longer side of the prism, when one of the short sides is directed square at a star, will reflect it horizontally through the axial tube. If such a telescope is used only as a transit-circle it will always lie east and west, with a motion of rotation round the axis for directing the prism to the proper altitude; and that motion is measurable on a sufficiently large graduated wheel, in the plane of the meridian, which makes a mural circle of it. And again, if this horizontal telescope is mounted on a vertical axis, so as to revolve in a horizontal plane, it becomes an altazimuth, the horizontal motion being in azimuth, and the rotation of the mirror giving the altitude as before. This enables the observer always to stand at the same level and look horizontally in the natural way.

Equatorial.—If the frame in which a telescope is set turns on an axis not vertical, but parallel to the earth's axis, that also gives an universal motion; the telescope can be pointed to a star and then clamped or screwed fast to the frame, and if the frame is turned round from east to west at the same rate that the earth turns the opposite way, the star will stay in the telescope. It is called an *Equatorial*, because each point in it then

moves parallel to the equator. That is the instrument used for gazing at the planets or moon, and sweeping the sky for new planets or comets, besides other purposes independent of the meridian. Therefore the largest object glasses that can be made perfect are generally used for equatorials. Herschel's great telescope was set in a frame which also turned azimuthally; and Lord Rosse's larger one, though it stands between stone piers, is capable of some sideway motion so as to be used equatorially, and can be kept pointed to a star for some time by machinery for the purpose.

In those heavy telescopes the speculum end of the tube stays on the ground. The lighter telescopes which are mounted equatorially are balanced on their middle: and they are often connected with a peculiar kind of clock which turns the frame at the proper rate, and so keeps the telescope pointed to the star without the observer having anything to do except himself to follow it. These clocks do not go by beats of a vibrating pendulum, which would be magnified by the telescope into leaps of the star; but they have either a revolving pendulum, or balls like the 'governor' of a steam-engine, or some equivalent contrivance to make the motion continuous.*

Heliostat.—This name is given to an instrument for making the sun apparently stand still, by a mirror kept revolving by a continuous motion clock like those just now referred to, so that the sun's rays may remain reflected in the same direction. For this purpose the mirror must turn only half as fast as the earth, because

* See 'Rudimentary Treatise on Clocks, &c.,' p. 212, Sixth Edition.

the angle of reflection moves as much as the angle of incidence (p. 373). This is used because it is more convenient than turning the whole apparatus, whatever it may be, which is used in connection with the solar observations for the time. A *Siderostat* is the same kind of instrument for the stars.

The following plan has been adopted for recording observations at once. Another continuous motion clock drives a barrel covered with paper, turning round (say) once in ten minutes, and at the same time advancing endways, by its pivot being made into a screw, so that a pencil fixed against it would draw a spiral line round the barrel. When the observer sees his transit he touches a pin, which by the common electric telegraph machinery makes a pencil strike the barrel and make a dot; which marks its own time, and the time at which every point on the barrel comes under the pencil is defined. The time of every observation, except mere gazing at the disc of a planet or moon or comet, is an essential part of it, and a clock is a necessary companion of almost every telescope; but I have written about that elsewhere.

There are many other contrivances and adjustments for securing accuracy of position, observation, and measurement, which cannot be described here. You will have some idea of the accuracy now attained from this:—a good observer can tell when a star crosses the wire of his telescope within the tenth of a second by the clock, which is always placed so that he can both see and hear it, and count the beats from the minute and second which he has looked at just before.

Note to Page 2.

On the Laws of Nature.

Although the discoveries of science are of no consequence to our religious faith and life, and therefore need not be revealed, it was most unlikely that a Creator of the world would leave men to infer his existence gradually from physical phenomena, or allow them to say that as he had not declared himself they may believe or disbelieve as they please. The vast majority of mankind must always be incapable of drawing proper inferences from physical phenomena, or even understanding them. Nor can any theory of creation be absolutely proved therefrom, for the simple reason that no theory at all admits of absolute proof, as soon as we get beyond the region of abstract mathematical truth; for even what we call ocular demonstration often requires qualification for mistakes and delusions.

When people ask such questions then, as whether the creation of the world and the maintenance of the present course of nature by a single self-existent Creator can be proved, and say they will believe it when it is proved, they forget that their own belief of every scientific theory, and of every historical event which is at all extraordinary, depends only on the balance of probabilities. The *à priori* improbability of many historical events is so great that nothing but a large concurrence of testimony, or manifestly abiding consequences of such events, would induce cautious men to believe them. At different times the probabilities that the history was true actually vary; i.e., the minds of men vary about them, according to some new evidence or arguments, discovered or invented on one side or the other; though of course the real truth all along remains the same. And about some histories competent judges are still so much divided that the probability may yet turn either way.

The most improbable event that any one can think of would

be credible to him if he saw it, and was certain there was no delusion. If he did not see it himself, the degree of secondary evidence that would satisfy him would depend upon the constitution of his mind. Some men would be satisfied by the testimony of a few of their own friends whom they considered quite credible and safe against delusion, while others would require a great deal more of such testimony, and others perhaps would believe none. To go a step farther, the truth of the narration of any improbable event manifestly does not depend upon the witnesses of it remaining alive. If their testimony was sufficient to convince other people while they were alive, it ought to be sufficient after they are dead, provided it has been properly preserved; though the transmission introduces fresh chances of error, unless some abiding consequences have gone along with it, which can only be accounted for by the truth of the original story. Still the historical events which we call most certain, are only so because the probability of the testimony for them so overbalances their *à priori* improbability that in our opinion it amounts to certainty.

Again, those laws of nature which we call certain are only so because they account for all the phenomena to which they are applicable better than any other hypothesis. Until this century nearly every philosopher felt certain that light was an emanation of particles; but they are all now equally certain of the contrary (see p. 101). In other words, the certainty of any physical cause is not absolute, but is merely our estimate of the probabilities in favour of one theory over another. We conclude without any doubt that gravity is universal, because the accepted theory of gravity accounts for every phenomenon within its reach, and there is no fact known against it. But even that cannot be pronounced absolutely certain. If Neptune had not been discovered, the hypothesis might have come to be believed, which actually was suggested, that gravity varies in its law of action at very great distances (p. 228), as if it took some time to travel. No one has the smallest doubt that the outstanding error in the lunar theory (p. 205) is due to some undiscovered disturbance, and not to any error in the received law of gravity; but nobody as yet can prove it.

In the same way the probabilities in favour of a single self-existing first cause, or Creator and maintainer of the world and all the forces in it and of what we call the laws of nature, have to be weighed against the probabilities for any other theory, by seeing what the two rival theories involve and really mean. The meaning of one of them is far from being realised by most people, and is evaded by its advocates in every possible way, as any one may see who reads their controversial or didactic writings, or converses with them. The fundamental theory of materialism, putting aside all the circumlocutory and evasive phrases in which it is disguised, is that all matter is self-existent *ab æterno,* together with the laws or courses of action which it follows. But it was remarked by Paley long ago, and by others before him, that 'the terms, "nature, powers of nature," 'and the like, are nothing but empty words, and signify merely 'that a thing usually comes to pass.' The question is, What makes it come to pass? The atheistic or materialistic theory requires some power for that purpose, just as much as the theistic; and there is no middle way between them. The theory of a single omnipotent Creator requires no explanation, and is undeniably sufficient to account for all the phenomena; but the other theory requires some examination, to ascertain its true scientific meaning.

To begin with the most universal laws of all, what is the meaning of gravity and inertia being 'inherent in matter'? for that is a favourite phrase with the materialists, as if the question were not, how did they come to be inherent? Moreover, the use of the word Matter in this way is a mere rhetorical artifice. Matter is not an unit, but is the sum of all the atoms in the universe. And the law of gravity being universal means that every atom in the universe attracts every other, according to the well-known law of the inverse square of the distance. But how did that infinity of atoms, separated by immeasurable distances, and constituting such a variety of substances, all come to agree that they would for ever attract each other and be attracted in that particular mathematical ratio? And when we say 'attract and be attracted,' we must remember that those again are only words, meaning that every atom will for ever try to approach every

other atom with a force varying inversely as their $(\text{distance})^2$. They may be unable to move the least towards each other, and yet they have agreed that they will always try. We may observe too that this force of attraction is quite different from, and indeed opposite to all similar forces which we are accustomed to or can imitate, except by complicated contrivances. The resistance of a spring, or of a weight pulled aside in a pendulum, increases with the distance instead of diminishing; and the commonest of all forces, the mere resistance of a weight to being lifted, does not diminish with any height we can reach. Although the inverse square of the distance is the natural and obvious law of emanation (p. 21) it is far from being an obvious or probable one *à priori* for attraction. Indeed we have seen (at p. 25) that one of the greatest philosophers of our time could hardly believe that there was not some mistake about it. So far is it from being self-evident or obvious as a necessary law of nature.

How then did the atoms scattered all over the infinity of space, millions and billions of miles apart, come to be more intelligent than Faraday, and agree to adopt this peculiar mathematical law of their behaviour towards each other? It is a mere evasion of the difficulty to suggest that it may be a result of some more general and comprehensive law. First, there is not the smallest evidence for that hypothesis; and secondly, if there were, we have only to transfer the same question to that more comprehensive law, and inquire how the atoms of the universe agreed to adopt it. For it is undeniable that either they did agree to adopt every law that they observe, i.e., resolved that they would always behave accordingly, or else that some external power has imposed it on them.

And this universal law of gravity is only one of an innumerable multitude of laws of nature which the different kinds of matter have agreed to follow. So far as we know, there are about sixty-three elements, or distinct elementary kinds of matter, unchangeable into each other. The number is of no consequence to the argument. There may be some yet undiscovered in the earth, and many in the stars, or it may turn out that some of those sixty-three are reducible to others. Again, it may be that they all have

the same kind of ultimate atoms, which have been somehow permanently combined into different kinds of 'molecules,' as they are called; or on the other hand, the ultimate atoms of the different elements may be different. But in any of those cases the question has equally to be answered, How did the atoms come to resolve that the elements which they compose should always and everywhere behave in a vast variety of different but constant ways in certain circumstances and combinations? And how did they combine into the various classes of molecules? No one doubts that the laws of their behaviour are prepared for them in circumstances and combinations which have never yet occurred perhaps anywhere, and certainly never to innumerable quantities of each class of atoms. We know that hydrogen gas behaves in some of the stars as it does here. Yet they are so far off that even their light takes many years to come. How did the atoms of that hydrogen resolve, by a kind of universal suffrage communicated from all that distance, to behave exactly like ours? We actually receive iron, and some other things, in meteors from enormous distances within or beyond the solar system, and it behaves here just as if it had been dug out of the next field, and smelted in our furnaces. It is no answer to say that all the solar system was probably one nebulous mass once. That only means that the constituents of the mass were much farther apart than they are now, being not then gathered into lumps called planets; and the question remains the same, How did all the atoms of iron, and everything else, when they were spread like gas into a nebulous sphere with a diameter larger than the orbit of Neptune, agree among themselves, and agree also with the atoms of all the other elements, how they would behave for ever in circumstances altogether different?

Even the atoms of the 'luminiferous æther,' which is supposed to be the vehicle of light and heat, and therefore filling all space, and infinitely exceeding even the infinite number of visible atoms more commonly called matter, must have similarly agreed among themselves, and with the atoms of all transparent bodies, how they were to behave with respect to each other, and as to the laws of their vibrations, which require the highest mathematical powers to investigate them. Yet they are supposed by the

materialistic theory to have settled for themselves the laws of reflection, refraction, spectroscopy, polarization, and the whole of the difficult science of optics, which nobody but the best mathematicians can understand—if they do yet completely.

All that we have thus far spoken of are the laws of matter without life. What are called organized bodies have yet another set of laws, different from and opposed to those of the very same matter when dead; prevailing even over gravity, as in the growing upwards of vegetables, great and small. How life began is entirely an unsolved problem scientifically, and it is equally unsolved how it is continued. All we know is the observed fact that living bodies have the power of continuing themselves for a certain time by constantly absorbing new particles and throwing off old ones; and the still more mysterious power of producing seeds, which will some day, after going through various dissimilar stages, grow into the likeness of their parents, which after a time die, and their atoms then fall back under their ordinary non-organic laws. For all these living things are composed of common atoms, which in their ordinary state have no power to do anything except to move under general and chemical attractions. So that they must have included in their original resolutions as to their behaviour, all the organic as well as non-organic laws, and must have invented all the various kinds of life also for themselves—if a Creator did not do it for them.

And it is only some of them which resolved to be capable of entering into or supporting life, such as oxygen, hydrogen, nitrogen, carbon, lime, phosphorus, and a few others in less degree. Others may be taken in accidentally, and pass out again without effect: others, if taken in at all, break up the living body and turn it back into a dead one, and some produce that effect on some bodies, but none at all on others. All these complicated varieties of action must have been settled for ever by agreement among the atoms at the beginning: e.g. all the atoms of carbon, oxygen, and nitrogen in the universe, must have agreed with all those capable of forming part of any human body, and some others, that whenever they should be combined in the proportions which form prussic acid, and a very small quantity

of such a combination should enter such a body, the others should forthwith refuse to go on any longer performing those functions which are called life, and leave the body to decay under another set of chemical laws.

But this is a very small part of what the atoms must have agreed to do for themselves, on the hypothesis of there being no superior power which ordained all their actions for them, and maintains them constantly. They must not only have resolved to obey the organic laws of life, but they must have resolved and known how to compose the infinite complications and contrivances of all living bodies. Let any one compare the immense sweep and multitude of such things embraced in one such book as Paley's 'Natural Theology,' with the little bits and fragments of conjectural explanations of a very few of them, on any spontaneous non-creative theory, which you can find in all the books together which modern materialists have brought forth. Yet Paley only professed to give a summary and superficial view of the most striking contrivances and adaptations of one thing in nature to another. Of all the foolish objections ever made by those who want to depreciate a book which they never try to answer, the most absurd is that it is 'superficial,' and that Paley only got his knowledge second-hand. He expressly said so himself. But if 'superficial' means wrong or inaccurate for the purposes of his argument, the book ought to be all the easier to refute; and yet they dare not try. If it does not mean wrong or inaccurate, it means that he only noticed facts which lie on the surface and are obvious to any one who will take the trouble to observe them, without condescending on the infinite number which he might have added.*

It is one of the distinctions between the works of men and of what we call nature, that there is no discoverable limit to the minuteness of the 'finish,' or composition of the latter; while we soon reach that limit in our own works. We say a house or a machine is built of bricks or stones or bits of metal; and we can go no further. A picture is composed of a definite number

* A new edition of it, corrected for the present state of science by the President of the College of Surgeons, has been published by the S.P.C.K.

of dabs of paint, not very small. But no microscope can reach the ultimate elements of which the compositions of nature are made up. And yet all the atoms which have entered into that composition have known exactly what to do in order to produce results of the utmost complication. There is no greater mistake than to talk of the 'simplicity of nature.' At p. 102 I mentioned that the retina of the eye, which used to be considered a mere membrane, is found to be a kind of brush when examined with strong microscopic power. And even that is far short of a full description. Those who have seen magnified pictures of it exhibited know that each hair of the brush has several parts, and things like cells at various places along its length, and that they all pass through a second membrane at a definite distance from the lower one in which they have their roots, besides other complications which I cannot describe, and of which no one yet knows the use. Nor can we be sure that still stronger magnification will not reveal further complications. Whether it does or not, the fact is beyond question, that a quantity of intelligent or unintelligent atoms have either combined themselves or been combined by some external power into all these complicated and yet constant forms, adapted to receive and to achromatize light, so as to produce a far more perfect image on the retina, and send it through the optic nerve to the brain, than we can manage with all our telescopes. And so we have to take our choice between the atoms having intelligently combined for that purpose, and doing so continually in every eye in the world, or being unintelligent and impotent themselves, but under the dominion of a Creator of sufficient intelligence and power to produce these results and innumerable others of the same kind.

Any one who is content to be put off from the fair examination of these two sole alternatives of creation by empty phrases about the 'potency of matter,' the 'immanence' or 'inherence of the laws of nature in matter,' and the like, wilfully surrenders his reason to deception. And any one who believes that the intelligence which produced the universe resides in the primary atoms which constitute matter, believes something far more improbable and incredible than the wildest superstition or the most absurd cosmogony ever invented. Yet there is no

intervening possibility between that hypothesis and a single intelligent Creator, who made all things and continually enforces all the laws of nature.

Whether they were sufficient to evolve from the primitive atoms all that now constitutes the universe, inanimate and animated, without any fresh creative interference, is a question quite distinct from the origin of those laws. And it is not a theological question, except as to the creation of man 'in the image of God' and with spiritual conditions on which physical science can throw no light whatever, even if it should make any real discovery as to his physical origin, which it certainly has not done yet. The theory of evolution without interference attributes, not a lower, but a higher degree of foresight, power, and intelligence to the author of the laws of nature, than the theory of occasional interference. For it is manifestly more difficult to invent machinery which will go on producing continually improved results for ever, than machinery which requires continually helping to do so by some fresh appliance. The hypothesis that such machinery was spontaneously generated by the unanimous concurrence of the atoms of the universe is the most outrageously absurd that ever was propounded under the name of 'science falsely so called.' Yet we may defy any materialist to show that that is not the true and simple meaning of his theory, however he may try to evade it by such phrases as I have quoted, or by refusing (as they often do) to meet that question at all, or to state their own theory distinctly.

Even if self-existing laws of nature, agreed on by the infinite congregation of atoms of the universe, were a rationally conceivable probability, there is still another thing which they must have designed themselves to do, and that is to produce results which satisfy the wishes of the animal creation, and our ideas of beauty. All the ingenious and continually amended and supplemented theories of 'selection' by animals for pleasure or beauty, or by accident, do nothing towards explaining that. What would the condition of the world have been if things which are fit to eat were not also good to eat? Suppose food had been as nasty as medicine. Will any rational man be satisfied with being told that it is by long usage that we have got to like our necessary food? What was the human race, and still more,

all the animal race, doing while they were learning to like it, if the time ever was when they did not? Were they all taking food as we do medicine? Animals now will starve rather than eat what they do not like, and so they would then, and would soon have starved out of existence.

Again, what kind of natural selection made all the natural phenomena, and what is called the face of nature, beautiful? Did the parliament of atoms resolve that they would so combine that all the trees of the earth, and flowers and fruits, hills and valleys, waves of the sea, streams and torrents, still lakes and running brooks, sunshine and moonlight and the stars, the clouds and the dew on the grass, the rainbow and the northern lights, thunder and lightning, the blaze of fire, the smell of many herbs and flowers, and of the earth itself, should be to our senses beautiful or pleasant, or what we call grand? The existence of exceptions, as of some stinking yet common herbs and other things, and especially of unwholesome smells, and a few ugly animals, and notably the one most like ourselves externally in other respects, at once refutes the theory thatwe admire the others merely from habit and from having grown into it in long ages. Yet I have heard that explanation seriously proposed by a distinguished scientific man as a reasonable solution of this difficulty of the materialistic theory. Such an attempt is equivalent to an admission that all these things, besides innumerable others, are inexplicable by any theory except that of a Creator of the world, who perpetually maintains the laws of nature and all the forces of the universe.

NOTE TO PAGE 106.

It should have been observed that if the luminiferous æther is so constructed that bodies can go through it without friction, their motion would not be resisted; and therefore planets' or comets' periods would not be shortened by it. It is demonstrable that the only resistance to a fish moving under water (though not to a ship which raises a wave) is that due to friction. We have no actual knowledge of any frictionless medium, but it is perfectly conceivable.

INDEX.

LONDON: PRINTED BY WILLIAM CLOWES AND SONS, STAMFORD STREET AND CHARING CROSS.

ity ce.	Apparent diameter of Sun there.	Heat and Light there.	Spheroid-icity.	Symbol.
5	..	..	0	☉
	32′	See p. 133	?	☾
33	120	6·9	?	☿
	44′	2	?	♀
	32′	1	$\frac{1}{298}$	⊕
3	21′	·45	?	♂
5,&c.	12	·08	..	(88)
5	6′ 6″	·037	$\frac{1}{17}$	♃
5	3′ 20″	·011	$\frac{1}{11}$	♄
5	1′ 42″	·003	$\frac{1}{10}$	♅
3	1′	·001	?	♆

241.

. . . 54,830
. . . 46,073
. . . 37,050
. . . 124,500

PUBLICATIONS

OF THE

Society for Promoting Christian Knowledge.

Most of these Works may be had in Ornamental Bindings, with Gilt Edges, at a small extra charge.

	Price. s.	d.
ANNALS OF HARTFELL CHASE. By Miss A. C. CHAMBERS, author of "Away on the Moorland," &c. With Three full-page Illustrations on toned paper. Crown 8vo.... *Cloth boards*	1	6
BRAG AND HOLDFAST. By EADGYTH, author of "The Snow Fort," &c. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards*	1	6
CRINGLEWOOD COURT. By F. SCARLETT POTTER, author of "Out-of-Doors Friends," &c. With Three full-page Illustrations on toned paper. Crown 8vo.................................. *Cloth boards*	2	6
DRIFTED AWAY: a Tale of Adventure. With Three full-page Illustrations on toned paper. Crown 8vo...... *Cloth boards*	2	6
ERLING; OR, THE DAYS OF ST. OLAF. By F. SCARLETT POTTER, author of "Heroes of the North," "Out-of-Doors Friends," &c. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards*	2	6
FAN'S SILKEN STRING. By ANNETTE LYSTER, author of "North Wind and Sunshine," &c. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards*	1	6
FOR FAITH AND FATHERLAND. By M. BRAMSTON, author of "Rosamond Ferrars," &c. With Three full-page Illustrations on toned paper. Crown 8vo.............................. *Cloth boards*	2	5
FRONTIER FORT (THE); OR, STIRRING TIMES IN THE NORTH-WEST TERRITORY OF BRITISH AMERICA. By W. H. G. KINGSTON, author of "Ned Garth," &c. With Three full-page Illustrations on toned paper. Crown 8vo.................. *Cloth boards*	1	6

	Price. s. d.
GIRLS OF BREDON (THE); AND MANOR HOUSE STORIES. By Mrs. STANLEY LEATHES. With Three full-page Illustrations on toned paper. Crown 8vo. ....*Cloth boards*	2 0
GREAT CAPTAIN (THE): an Eventful Chapter in Spanish History. By ULICK R. BURKE, M.A. With Two full-page Illustrations on toned paper. Crown 8vo..................*Cloth boards*	2 0
GUINEA GOLD; OR, THE GREAT BARRIER REEF. By C. H. EDEN, author of "Philip Vandeleur's Victory," &c. With Three full-page Illustrations on toned paper. Crown 8vo. *Cloth bds.*	1 6
HARRY PRESTON; OR, "TO HIM THAT OVERCOMETH." A Story for Boys. By the author of "Ellen Mansel." With Three full-page Illustrations on toned paper. Crown 8vo. ...*Cloth boards*	1 0
HARVEY COMPTON'S HOLIDAY. By the author of "Percy Trevor's Training," "Two Voyages," &c. With Three full-page Illustrations on toned paper. Crown 8vo............... *Cloth boards*	1 6
HEROES OF THE NORTH; OR, STORIES FROM NORWEGIAN CHRONICLE. By F. SCARLETT POTTER, author of "Erling" "Cringlewood Court," &c. With Three full-page Illustrations on toned paper. Crown 8vo. ....*Cloth boards*	2 6
HIDDEN WORKINGS. By Miss H. R. RUSSELL, author of "Muriel," &c. With Three full-page Illustrations on toned paper. Crown 8vo.. *Cloth boards*	2 0
HOME OF THE WOLVERENE AND BEAVER (THE); OR, FUR-HUNTING IN THE WILDS OF CANADA. By C. H. EDEN, author of "Australia's Heroes," &c. With Three full-page Illustrations on toned paper. Crown 8vo. ....*Cloth boards*	2 6
KING'S WARRANT (THE). A Tale of Old and New France. By A. H. ENGELBACH, author of "Lionel's Revenge," &c. With Three full-page Illustrations on toned paper. Crown 8vo. *Cl. bds.*	2 6
KITTY BLIGH'S BIRTHDAY. By ALFRED H. ENGELBACH, author of "Lionel's Revenge," &c. With Three full-page Illustrations on toned paper. Crown 8vo..........................*Cloth boards*	1 6
LEFT IN CHARGE. Being the History of My Great Responsibility. By AUSTIN CLARE, author of "The Carved Cartoon," &c. With Three full-page Illustrations on toned paper. Crown 8vo. ....*Cloth boards*	1 6

	Price. s. d.
LITTLE BROWN GIRL (THE). A Story for Children. By ESMÉ STUART. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards*	2 6
MATE OF THE "LILY" (THE); OR, NOTES FROM HARRY MUSGRAVE'S LOG-BOOK. By WILLIAM H. G. KINGSTON, author of "Owen Hartley," &c. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards*	1 6
MIMI: A STORY OF PEASANT LIFE IN NORMANDY. By ESMÉ STUART, author of "The Little Brown Girl," &c. With Three full-page Illustrations on toned paper. Crown 8vo. *Cloth bds.*	2 6
MRS. DOBBS'S DULL BOY. By ANNETTE LYSTER, author of "Fan's Silken String," "North Wind and Sunshine," &c. With Three full-page Illustrations on toned paper. Crown 8vo. *Cloth bds.*	2 6
NED GARTH; OR, MADE PRISONER IN AFRICA. A Tale of the Slave Trade. By W. H. G. KINGSTON, author of "Owen Hartley," "Two Shipmates," &c. With Three full-page Illustrations on toned paper. Crown 8vo......................... *Cloth boards*	2 6
NORTH WIND AND SUNSHINE. By ANNETTE LYSTER, author of "Mrs. Dobbs's Dull Boy," &c. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards*	2 6
NOT A SUCCESS. By the author of "Our Valley," "The Children of Seeligsberg," &c. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards*	1 6
OUR VALLEY. By the author of "The Children of Seeligsberg," &c. With Three full-page Illustrations on toned paper. Crown 8vo.. *Cloth boards*	2 6
OWEN HARTLEY; OR, UPS AND DOWNS. A Tale of the Land and Sea. By WILLIAM H. G. KINGSTON, author of "The Settlers," &c. &c. With Five full-page Illustrations on toned paper. Crown 8vo..................................... *Cloth boards*	2 6
PERCY TREVOR'S TRAINING. By the author of "Two Voyages," &c. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards*	2 6
PHILIP VANDELEUR'S VICTORY. By C. H. EDEN, author of "Australia's Heroes," "The Fifth Continent," &c. With Three full-page Illustrations on toned paper. Crown 8vo. *Cloth boards*	2 6

Price.
s. d.

PIANO IN THE ATTIC (THE). By ANNETTE LYSTER, author of "Fan's Broken String," &c. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards* 1 6

RECLAIMED. A Tale. By the Rev. A. EUBULE-EVANS. With Three full-page Illustrations o toned paper. Crown 8vo. *Cloth boards* 2 6

ROSEBUDS. By the author of "Our Valley," &c. With Three full-page Illustrations on toned paper. Crown 8vo. *Cloth bds.* 2 6

ROYAL BANNER (THE). A Tale of Life Before and After Confirmation. By AUSTIN CLARE, author of "The Carved Cartoon," &c. With Three full-page Illustrations on toned paper, Crown 8vo. .... *Cloth boards* 2 6

SNOWBALL SOCIETY (THE) A Story for Children. By M. BRAMSTON, author of "Rosamond Ferrars," &c. With Three full-page Illustrations on toned paper. Crown 8vo....... *Cloth boards* 2 6

SNOW FORT AND THE FROZEN LAKE (THE); OR, CHRISTMAS HOLIDAYS AT POND HOUSE. By EADGYTH. With Three full-page Illustrations on toned paper. Crown 8vo. *Cloth boards* 2 6

STORIES FROM ITALIAN HISTORY. By B. MONTGOMERIE RANKING. With Two full-page Illustrations on toned paper. Crown 8vo.. *Cloth boards* 1 6

TWO VOYAGES, AND WHAT CAME OF THEM. By the author of "Motherless Maggie," &c. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards* 2 0

WILFORD FAMILY (THE); OR, HERO-WORSHIP IN THE SCHOOLROOM. By EADGYTH. With Three full-page Illustrations on toned paper. Crown 8vo. .... *Cloth boards* 1 6

DEPOSITORIES:

NORTHUMBERLAND AVENUE, CHARING CROSS, S.W.;

4, ROYAL EXCHANGE, E.C.; AND 48, PICCADILLY, W.